Physics 326a-2013  Walter F. Smith
Assignment 1

Due:  Friday 9-13-13, 1:20 pm
Reading:  Lab Manual B1-B12, C4-C7

Assigned exercises:
Note: for all problems in this course, limit your resistors to the range 20 Ω to 1 MΩ (500 Ω to 250 kΩ is best, when possible), and capacitances for non-polarized capacitors (all the capacitors used in these problems are non-polarized) to the range 20 pF to 1 μF (100 pF to 50 nF is best, when possible).

You are encouraged to work together with your classmates, in small groups, on problem sets. However, for your own good, avoid situations in which you are either contributing too little or too much to the collaboration.

1.1 a. Show that if \( A = BC \), then \( A = BC \), where \( A \), \( B \), and \( C \) are complex numbers, and \( A, B, \) and \( C \) are their magnitudes. Hint: represent each complex number in polar form, e.g. \( A = Ae^{iθ} \). Using this hint, this is an extremely easy problem.

b. Show that if \( A = \frac{B}{C} \), then \( A = \frac{B}{C} \).

c. Show that if \( A = B + C \), then it is not usually true that \( A = B + C \).

1.2 On p. C6 of the lab manual, I show that, for a high-pass filter, \[ \left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{1}{\sqrt{1 + \left( \frac{f_{HI}}{f} \right)^2}} \], where \( f_{HI} \) is the \( f_{3dB} \) of the high-pass filter. For a low-pass filter, show that \[ \left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{1}{\sqrt{1 + \left( \frac{f}{f_{LO}} \right)^2}} \], where \( f_{LO} \) is the \( f_{3dB} \) of the low-pass filter.

1.3 Consider the series combination of a low pass filter and a high pass filter shown here (the same as what you will make in lab). Show that, at \( f = 7.59 \) kHz, \[ \left| \frac{V_{OUT}}{V_{IN}} \right| = 0.58 \]. (This frequency happens to be the geometric mean of the \( f_{3dB} \) of the low pass and that of the high pass.) Hints: Do problems 1.1 and 1.2 first. Don’t forget about loading effects. Also, the algebra gets messy quickly. As soon as you have expressions for \( Z_{IN} \) and \( Z_{OUT} \), go ahead and substitute numerical values. Then use a calculator that can handle complex numbers or Mathematica to evaluate \( \frac{V_{OUT}}{V_{IN}} \). The Mathematica command to find the
magnitude of a complex number is Abs[number]. You may need to make use of the ComplexExpand function, which forces Mathematica to assume that all symbolic quantities are real, unless i is explicitly included (which you do by typing I or esc-ii-esc).

1.4 How would you make a filter with resistors and capacitors to give the response shown here? (A qualitative answer is fine – you don’t need to give the values for the components.)

1.5 a. For a low-pass filter, show that \( \frac{V_{\text{out}}}{V_{\text{in}}} \) drops by a factor of ten when the frequency is increased by a factor of ten, for frequencies well above \( f_{3\text{dB}} \). \textit{Hint: Do problem 1.2 first.}

b. For a high-pass filter, show that \( \frac{V_{\text{out}}}{V_{\text{in}}} \) drops by a factor of ten when the frequency is decreased by a factor of ten, for frequencies well below \( f_{3\text{dB}} \).

1.6 Come up with a mnemonic phrase for the resistor color code (Black Brown Red Orange Yellow Green Blue Violet/Purple Gray White) e.g. “Bad Borg Raid Our Young Galaxy Before Vaporizing Good Walter” (an entry from a few years back).