Physics 214b-2008  Walter F. Smith  Assignment 2

Due: 4 pm Wednesday 2-6-08

Reading: Section 2.1

Exercises

All problems are group problems, unless otherwise marked; you are encouraged to work in small groups on these. For the individual problems, you are not allowed to consult any other students, but you may ask me for help.

2A. Individual problem  Starting from \( p = \gamma mu \) and \( E = \gamma mc^2 \), derive \( E^2 = p^2 c^2 + m^2 c^4 \).

2B.  
   a. Show that \((1 + x)^\beta \simeq 1 + \beta x\), for \( x << 1 \).
   b. Memorize and eat the above relation. You will use it all the time in the rest of your physics life.
   c. Noting that \( \beta \) need not be an integer in the relation from part a, use it to show that \( K = \gamma mc^2 - mc^2 \) reduces to the classical expression for low velocities.

2C. The Planck distribution, part one.
   a. You are given the chance to play a peculiar game of dice. To play, you would have to pay $10. You would then throw a die once, and win the following amounts based on the result: For a 1 or 2, you win $1. For a 3 or 4, you win $2. For a 5, you win $10. For a 6, you win $25. You’re told up front that the die being used is weighted, so that the probability of each of the six faces coming up is not the same. Before you have to decide whether to play, you are allowed to test the die. You roll it quite a few times, with the following results:
      1: 359 times
      2: 353 times
      3: 403 times
      4: 516 times
      5: 108 times
      6: 100 times
   
      Based on these results, what is the average amount that you would expect to win from one roll of the die?
   
      Will you, on average, make a profit from playing this game, given that it costs $10 to play each round?
   b. As we discussed in class, Planck showed that the probability of a normal mode being excited to a level of \( n \) quanta is proportional to \( e^{-nh\nu/k_BT} \), where \( k_B = 1.38 \pm 23 \) J/K is Boltzmann’s constant. For example, to excite an electromagnetic standing wave to \( 3h\nu \) of energy, the probability is proportional to \( e^{-3h\nu/k_BT} \).
      Explain why the average number of quanta (photons for electromagnetic standing waves, phonons for atomic vibrations) is then given by
      \[
      \langle n \rangle = \frac{\sum_{n=0}^{\infty} n e^{-nh\nu/k_BT}}{\sum_{n=0}^{\infty} e^{-nh\nu/k_BT}}.
      \]

Townsend
1.10
1.11 Individual problem
1.12
1.16 Individual problem
1.44 (Note that we will not discuss the principle of least time in class; I expect you to read about it in ch. 1.)