The eigenstates of $\hat{S}_x$ are:

\[
\chi_{x\pm} = \frac{1}{\sqrt{2}} (\chi_+ + \chi_-) \quad \text{"spin right"}
\]

\[
\chi_{x\pm} = \frac{1}{\sqrt{2}} (\chi_+ - \chi_-) \quad \text{"spin left"}
\]

\[
\chi_+ = \frac{1}{\sqrt{2}} (\chi_{x+} + \chi_{x-}) \quad \text{"spin up"}
\]

\[
\chi_- = \frac{1}{\sqrt{2}} (\chi_{x+} - \chi_{x-}) \quad \text{"spin down"}
\]

\[\Psi(\vec{r}_1, \vec{r}_2, t)\] obeys

\[\hat{\mathcal{H}} \Psi(\vec{r}_1, \vec{r}_2, t) = i\hbar \frac{\partial \Psi(\vec{r}_1, \vec{r}_2, t)}{\partial t}\]

Where

\[\hat{\mathcal{H}} = -\frac{\hbar^2}{2m_1} \nabla_{\vec{r}_1}^2 - \frac{\hbar^2}{2m_2} \nabla_{\vec{r}_2}^2 + V(\vec{r}_1, \vec{r}_2, t)\]

If $V$ has no explicit time dependence, we can do the same separation of variables as before:

\[\Psi(\vec{r}_1, \vec{r}_2, t) = \varphi(\vec{r}_1) \varphi(\vec{r}_2) e^{-iEt/\hbar}\]

Where

\[\varphi(\vec{r}_1) \varphi(\vec{r}_2) = E \varphi(\vec{r}_1) \varphi(\vec{r}_2)\]

For now, set aside considerations of spin.

Suppose particle 1 is in the state $\varphi_\alpha(\vec{r}_1)$ and particle 2 is in the state $\varphi_\beta(\vec{r}_2)$. Then, if the particles are distinguishable,

\[\Psi(\vec{r}_1, \vec{r}_2) = \varphi_\alpha(\vec{r}_1) \varphi_\beta(\vec{r}_2)\]

Example: Two particles in infinite well

\[\varphi_\alpha(x_1) = \sqrt{\frac{2}{L}} \sin \frac{\pi x_1}{L}\]

\[\varphi_\alpha(x_2) = \sqrt{\frac{2}{L}} \sin \frac{\pi x_2}{L}\]

\[\Psi(x_1, x_2) = \frac{2}{L} \sin \frac{\pi x_1}{L} \sin \frac{\pi x_2}{L}\]
Indistinguishability

However, if the two particles are identical (e.g., two electrons), then swapping them should have no physical consequence.

Define the exchange operator: \( \hat{P}_{12} \Psi(\vec{r}_1, \vec{r}_2) = \Psi(\vec{r}_2, \vec{r}_1) \)

Indistinguishability \( \Rightarrow \hat{P}_{12} \Psi(\vec{r}_1, \vec{r}_2) = e^{i\phi} \Psi(\vec{r}_1, \vec{r}_2) \)

Overall phase factor:

Thus, no physical consequence.

Swapping particles twice can have no effect: \( \hat{P}_{12}^2 \Psi(\vec{r}_1, \vec{r}_2) = (e^{i\phi})^2 \Psi(\vec{r}_1, \vec{r}_2) = \Psi(\vec{r}_1, \vec{r}_2) \)

\( \Rightarrow (e^{i\phi})^2 = 1 \Rightarrow \hat{P}_{12} \Psi(\vec{r}_1, \vec{r}_2) = \pm \Psi(\vec{r}_1, \vec{r}_2) \)