One can find $\Theta$ for $M \neq 0$, but it is lengthy. (Handout coming.) It turns out that one still finds $L^2 = h^2 l(l+1)$

$$\Rightarrow * \left| m_e \right| \leq \ell *$$

$m_e$ orbital angular momentum number

$m$ azimuthal quantum number

Combining $\Theta$ with $\Phi$ and normalizing

$$\left( \int_0^{2\pi} \int_0^\pi \Theta \sin \theta \, d\theta \, d\phi \right) \text{ gives the }$$

"Spherical harmonics" $Y_{l,m}$. For example,

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}} \quad Y_{1,0} = \sqrt{\frac{3}{4\pi}} \sin \theta \quad Y_{1,\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \sin \theta \, e^{\pm i\phi}$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}} \left( 3 \cos^2 \theta - 1 \right)$$

Note: the radial equation: $V = -\frac{Ze^2}{4\pi \varepsilon_0 r}$ (hydrogenic atom)

$$-\frac{\hbar^2}{2m} \left[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) \right] + \frac{l(l+1)}{2mr^2} R = \frac{Ze^2}{4\pi \varepsilon_0 r} R$$

$$u = R\,r$$

$$\Rightarrow -\frac{\hbar^2}{2m} \frac{d^2 u}{dr^2} + \left[ \frac{l(l+1)\hbar^2}{2mr^2} - \frac{Ze^2}{4\pi \varepsilon_0 r} \right] u = Eu$$

like 1D TISEQ with

$$V_{\text{eff}} = \frac{l(l+1)\hbar^2}{2mr^2} - \frac{Ze^2}{4\pi \varepsilon_0 r}$$

$$L = \frac{L^2}{2I} = K_{\text{rot}}$$

but it is more convenient to think of this as an effective potential energy, the "centrifugal barrier"