Physics 213-2011 Coverage for Exam 2

Reading:
Chapter 4: 4.1-4.6
Chapter 5: All
Chapter 6: 6.1-6.5

Class sessions: Last part of 9-19-11 through 10-26-11

Assignments: 4-7

Topics (topics in boldface are somewhat more important, but don’t ignore the others):

Damped driven harmonic oscillator:
- Relation between oscillatory force amplitude and amplitude of motion of support point
- DEQ of motion
- steady state solution:
  - response is at the angular frequency of the drive, not \( \omega_0 \)
  - be able to quickly write down and understand expressions (in complex exponential format) for and sketch position, velocity, and acceleration
- Dependence of amplitude on drive frequency:
  - be sure to have this expression on your equation sheet
  - be able to sketch the graph
  - awareness that peak position is slightly below \( \omega_0 \)
- Understand definition and meaning of \( \delta \), and how it depends on \( \omega_d \)
  - be sure to have this expression on your equation sheet
  - be able to sketch the graph
  - awareness that the inflection point (which is where \( \delta = \pi / 2 \) ) is at exactly \( \omega_0 \)
  - awareness that low Q causes “smearing” but doesn’t change asymptotes

What happens when multiple sinusoidal drive forces are applied

Power resonance curve
- Be able to sketch qualitatively, to explain qualitatively why it goes to zero for \( \omega_d \to 0 \) and for \( \omega_d \to \infty \), and why it has a maximum near \( \omega_0 \).
- know that FWHM = \( \gamma = \omega_0 / Q \)
- know that peak is at exactly \( \omega_0 \)
- Be able to calculate the power dissipated over a cycle or part of a cycle
- General solution: be able to “prove” (as we did in class) that this is the sum of the steady state solution and the general solution for the damped, undrived oscillator. Make sure you understand the meaning of all the symbols in the general solution, and which of them are affected by the initial conditions.

Damped driven series RLC oscillator, including isomorphism with mechanical oscillator

Beats:
- have the equation for \( x \) and definitions of \( \omega_{av} \) and \( \omega_e \) on your equation sheet
- meaning of beat frequency
- be able to sketch the beat behavior

Coupled oscillators (symmetric and asymmetric, with equal or unequal masses)
- How the physical coupling (e.g. the spring) leads to coupled differential equations for the motion of the objects in the system

Normal modes
- What the term “normal mode” means
  - Fact that energy that is put into one normal mode stays there (rather than being exchanged to a different normal mode)
- Fact that an arbitrary state of the system can be expressed as a superposition of normal modes of different amplitudes and phases
- Why we observe beats if we excite two normal modes with equal amplitudes
- The normal mode coordinates \( s_p \) and \( s_b \)
How to set up the eigenvalue equation for a system of $N$ coupled oscillators (which might include compound pendula, e.g. fig. 6.P.6)

How to then set up the characteristic equation
How to then find the eigenvalues
How to then find the eigenvectors
Meaning of the terms “eigenvalue equation”, “eigenvalue”, “eigenvector”

Matrices
- Multiplying
- Finding determinants
- Converting back and forth between systems of linear equations and matrix equations

Hilbert space
- As applied to normal mode expansions: any possible set of initial positions for the masses in the system, given zero initial velocities for all the masses, is represented by a point in this space. The vectors corresponding to initial configurations of pure normal mode states are called “eigenvectors”. The normalized eigenvectors have length 1, and act as “unit vectors”

Definition of the inner product

Bra-ket notation
- A ket represents a vector in Hilbert space
- How to convert from a ket to a bra
- Connection between kets and column vectors, and between bras and row vectors
- How to take an inner product

Normal mode analysis (for systems where all masses are equal, but the couplings aren’t necessarily symmetrical, and there may be more than two masses)
- Any behavior of a system of coupled oscillators can be expressed as a superposition of normal modes, each with its own amplitude and phase
- How this “normal mode expansion” way of thinking contains the same information as the more conventional way of thinking (in which we consider the position and velocity of each object in the system)
- Fact that each of the “basis vectors” in the expansion is orthogonal to each of the others.
- How to normalize an eigenvector

How to find the complex coefficients in the normal mode expansion given the initial positions and velocities.
How to then predict the behavior of the system for later times
Analogies between the normal mode expansion for the important special case of zero initial velocities and the expansion of an ordinary two-dimensional vector in terms of its components, including the analogy for how to find the expansion coefficients given the unit vectors and the vector of interest.

Driven coupled oscillators
- Each normal mode responds as an independent harmonic oscillator.
- For a symmetric, two-oscillator system, the effective drive amplitude is reduced by $1/\sqrt{2}$
- Driving a pendulum by moving the support point

Misc.
- How changing the length of a spring (e.g. by cutting) affects its spring constant
- What a wavenumber is, and how it’s related to the wavelength
- Kronecker delta function

General:
- Be solid on all topics covered in lecture and in the problem sets
- The best way to study is to study your lecture notes and the online lecture summaries and to work additional problems; this is more effective than re-reading the same passages in the book over and over

Be especially solid on all topics which appeared in starred boxes