Physics 213a-2011  Class 9  Monday 9-19-11  Summary

**Damped electrical oscillator**

\[ E = E_0 e^{-\gamma t} \]

- For timescales \( \gg \) period, \( E = E_0 e^{-\gamma t} \)

**Driven oscillators**

One way to provide a driving force:
move the support point:

\[ F_{\text{drive}} = +k x_c \]

To start, we consider a sinusoidal drive:

\[ F_{\text{drive}} = F_0 \cos \omega dt \]

Newton's 2nd law → \( \ddot{x} + \gamma \dot{x} + \omega_0^2 x = F_0 \cos \omega dt \)

\[ \ddot{z} + \gamma \dot{z} + \omega_0^2 z = F_0 e^{i \omega t} \]

Guess: \( z \equiv C e^{i \omega t} \)

\[ C \left( -\omega_0^2 + \gamma i \omega_0 + \omega_0^2 \right) = \frac{F_0}{m} \]
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Commutation of Re

Re commutes with $\frac{d}{dt}$: $Re \frac{dz}{dt} = \frac{d}{dt} Re z$

Re commutes with multiplication by a real number: $Re bz = b Re z$

Re does not commute with multiplication by a complex number: $Re C_1 C_2 \neq Re C_1 \cdot Re C_2$

A different way of solving the DEQ

Often, it's easier to solve the complex form of the DEQ:

$\dot{x} = -\omega_0^2 x$

$x = A \cos (\omega_0 t + \phi)$

Write complex version

$\dot{z} = -\omega_0^2 z$

$solve \rightarrow z = Ce^{i\omega_0 t} = Ae^{i(\omega_0 t + \phi)}$

$z = x + i b$

Review of complex numbers

A = 1c = $\sqrt{a^2 + b^2}$

$\varphi = \tan^{-1} \frac{b}{a}$

$A = a \cos \varphi$

$b = a \sin \varphi$

$C = a + ib = Ae^{i\varphi}$

- Complex conjugate $C^* = a - ib$ (replace each $i$ by $-i$)
- $CC^* = 1c^2$
- Multiplying by $e^{i\alpha}$ rotates complex plane vector counterclockwise by $\alpha$