Plane waves + MEQ + vacuum

Non-linear waves

For waves travelling in a medium, at very high amplitudes the DEQs become nonlinear → no longer get simple superposition

Example: frequency-doubling crystal (highly idealized description)

Model PE for electron

Excite with sinusoidal EM wave, model as harmonic oscillator

Can Fourier synthesize the non-sinusoidal response by summing responses at \( \omega, 2\omega, 3\omega \)...

⇒ electron radiates at \( \omega, 2\omega, 3\omega \)...

⇒ Get some photons out at \( 2\omega \)!

The superposition principle for traveling waves

The wave eqn is a linear DEQ ⇒ can superpose multiple solns. ⇒ For example:
Another example of non-linear waves: two-photon absorption

Irradiate molecule with photons of half the required energy. If intensity is extremely high, can essentially absorb two photons at once & cause the electron to jump to the higher level.

Can localize where the intensity is high enough by the intersection of two laser beams or by the “focussing” effect of a sharp metal tip.

The Poynting vector
\[ S = \frac{1}{2} \mu_0 \mathbf{E} \times \mathbf{B} \]
points in the direction of propagation.

Section 9.10 → |I|² = intensity

Traveling sinusoidal waves
\[ y = A \sin (k(x - v_p t)) = A \sin (kx - \omega t) \]

Phase velocity \[ v_p = \frac{\omega}{k} \]

Conventional way to write