Fourier Transforms

Important example: The Gaussian

\[ y(x) = A e^{-\frac{(x-x_0)^2}{2\sigma^2}} \]

For \( x_0 = 0 \)

\[ \Rightarrow y(k) = A_0 e^{-\frac{k^2}{(2\sigma^2)}} \]

To create a localized pulse in \( x \), with small FWHM, need a wide distribution of \( k \)'s \( \leftrightarrow \) Heisenberg uncertainty principle

Electromagnetic waves in vacuum

Are electromagnetic fields of the form

\[ \vec{E} = E(x,t) \hat{j}, \quad \vec{B} = B(x,t) \hat{k} \]

consistent

"Plane wave" with Maxwell's equations?

Faraday's Law + Plane wave

\[ \frac{\partial \vec{E}}{\partial x} = -\frac{\partial \vec{B}}{\partial t} \quad (A) \]

Ampère's Law + Plane wave

\[ \frac{\partial \vec{B}}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (B) \]

The Wave equation

\[ \frac{\partial^2 \vec{E}}{\partial t^2} = V_p^2 \frac{\partial^2 \vec{E}}{\partial x^2} \]

\[ V_p = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]

Solutions are \( E(x-V_p t) \) \& \( E(x+V_p t) \)

A "rigid" wave of any shape traveling right at speed \( V_p \)

A left-traveling wave