Windowing

The measurement window $T$ ordinarily doesn't match the periodicity of the signal, but the process of Fourier analysis assumes that it does. This leads to sharp corners in the assumed waveform:

The sharp corners introduce spurious high-frequency Fourier components. To avoid these, we multiply the signal by a windowing function before doing the Fourier analysis, as shown to the right:

The resulting function has no sharp corners, eliminating the spurious high frequency components. The downside of this process is a widening of the Fourier peaks, and some loss in accuracy of the peak heights.
**Fourier Transforms**

**Fourier analysis:**

A function $y(t)$ with periodicity $T$

$$y(t) = \sum_{n=-\infty}^{\infty} C_n e^{i\omega_n t} \quad C_n = \frac{1}{T} \int_{0}^{T} e^{-i\omega_n t} y(t) \, dt$$

$$\omega_n = \frac{n \frac{2\pi}{T}}$$

We can Fourier analyze a non-periodic function by allowing $T \to \infty$

**Problem 8.4**

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega) e^{i\omega t} \, d\omega \quad Y(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} y(t) \, dt$$

A Fourier Transform pair

$Y(\omega)$ is the "Fourier transform" of $y(t)$.

$y(t)$ is the inverse Fourier transform of $Y(\omega)$.

**Important example: The Gaussian**

$$y(x) = A e^{-(x-x_0)^2/2\sigma^2}$$

For $x_0 = 0$

$$\Rightarrow y(k) = A \sigma e^{-k^2/(2\sigma^2)}$$

Book $\Rightarrow$ FWHM $= 2.35\sigma$