Experimental Fourier Analysis

The experimenter acquires data during a time window $T$, & assumes the signal has periodicity $T$. Then $y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(\omega_n t + \phi_n)$

1. The "frequency resolution" (spacing between the $\omega$'s, divided by $2\pi$) = $\frac{1}{\Delta}$

2. For accurate sampling, need $\frac{1}{\Delta} \geq 2f_{\text{signal}}$

Maximum wiggliness

Where $f_{\text{signal}}$ is the frequency of the highest frequency sinusoid in the original signal $y(t)$. 
Experimental Fourier Analysis (cont.)

3. Aliasing

What if $f_{\text{sig}} > \frac{f_{\text{sample}}}{2}$?

$\frac{f_{\text{sig}}}{2} = \frac{1}{T}$

$\Delta f_{\text{Apparent}} = f_{\text{sig}} - f_{\text{sample}}$

Example: Measurement window $T=0.1$ s $\Rightarrow$ Frequency resolution $f_i = \frac{1}{T} = 10 \text{ Hz}$

Sampling rate $f_{\text{sample}} = 1000 \text{ Hz}$

Signal frequency $f_{\text{sig}} = 1010 \text{ Hz} = f_i + f_{\text{sample}}$

$\Rightarrow f_{\text{Apparent}} = f_{\text{sig}} - f_{\text{sample}} = 10 \text{ Hz}$

$\Rightarrow$ Can add or subtract $f_{\text{sample}}$ to the frequency of a sinusoid without changing the Fourier spectrum.

To avoid aliasing: use an anti-aliasing filter!

Typical measurement chain:

4. Conventional ordering of frequencies in the "Digital Fourier Transform"

Continuous function: $y(t) = \sum_{n=-\infty}^{\infty} C_n e^{i\omega n t}$

Discretely-sampled function: restrict the range of frequencies in the sum to $-f_{\text{max}}$ to $+f_{\text{max}} = \frac{f_{\text{sample}}}{2}$