The beaded string

\[ T \]

\[ m \]

\[ \alpha \]

\[ \rightarrow \text{effective spring constant for each} \]

\[ \text{string is } \frac{T}{\alpha} \]

\[ \psi \frac{\omega_A^2}{2} \begin{pmatrix} \ldots & 0 & 0 & -1 & 2 & 1 & \ldots \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \end{pmatrix} = \omega_n^2 \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \end{pmatrix} \]

where \( \omega_A = \sqrt{\frac{2T}{ma}} \)

and we have used the normal mode guess:

\[ y_j = \text{Re} z_j \quad z_j = y_j e^{i\omega t} \]

\[ \frac{\omega_n^2}{2} \begin{pmatrix} -Y_{j-1} + 2Y_j - Y_{j+1} \end{pmatrix} = \omega_n^2 Y_j \]

But for this system there is an easier way:

Guess the eigenvectors!

Based on homology of the 2-bead system w/2-pendulum system, & an observation of a continuous string, we make a...

Standing wave guess:

\[ Y_j = A_n \sin \frac{2\pi}{\lambda_n} x_j \]

where \( x_j = j \alpha \), and (because the string must go to zero at both ends) \( \lambda_n = \frac{2L}{n} \quad n=1,2,\ldots \)

Plugging this in to (A)

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\[ \rightarrow \text{Our guesses work if } \omega_n = \sqrt{\frac{2 T}{m}} \alpha n \]

where the wavenumber \( k_n = \frac{2\pi}{\lambda_n} \)

In principle, we could solve this using the methods we've just been studying (set determinant of the matrix in characteristic equ to zero, etc.)