Rotation

Angular

\[ \Theta \]

\[ \Delta \Theta \]

\[ r \Delta \Theta = s \]

tangential displacement,
a.k.a. arclength

\[ s = \frac{dx}{dt} \]

tangential component
of velocity

\[ \alpha = \frac{d\omega}{dt} \]

tangential component
of acceleration

Homology between rotation & linear motion

The relationships between \( \Delta \Theta, \omega, \) and \( \alpha \) are exactly the same as those between \( s, v, \) and \( a. \)

For constant angular acceleration, can use the same equations as for constant linear acceleration.

Acceleration, just by changing symbols:

\[ v = v_0 + at \]

\[ s = v_0 t + \frac{1}{2} at^2 \]

\[ r^2 = v_0^2 + 2as \]

constant linear acceleration

\[ \omega = \omega_0 + \alpha t \]

\[ \Delta \Theta = \omega_0 t + \frac{1}{2} \alpha t^2 \]

\[ \omega^2 = \omega_0^2 + 2\alpha \Delta \Theta \]

constant angular acceleration

Torque

A force works better for producing angular acceleration if it is applied far from the axis of rotation and perpendicular to the radius vector drawn from the axis of rotation to the point of force application.

\[ F \cdot \theta \]

only the tangential component of \( F \) acts to create rotation.

\[ \tau = \sum F \sin \theta \]

where \( \theta \) is the angle between \( F \) and \( \tau. \)

\[ \sum \tau = I \alpha \]

For a rigid body, where \( I \) is the moment of inertia

\[ I = \sum m_i r_i^2 \]

plays the role of "angular mass"