Damped, driven oscillators

At low speeds (typ. for oscillators) \( F_{\text{drag}} = -b \frac{dx}{dt} \)

\( x_c = \frac{A}{m} \cos(w_d t) \)

Similarly, observations \( \Rightarrow S(w_d) \)

\( S = 0 \) at \( w_d \approx 0 \)

\( \Rightarrow x(t) = A(w_d) \cos[w_d t - S(w_d)] \)

You'll show on next homework that this is a sol'n to the DEQ, and you'll determine the forms of \( A(w_d) \) and \( S(w_d) \). For now, we'll just look at the results:

\[ \begin{align*}
Q &= 4 \\
Q &= 2 \\
Q &= 1
\end{align*} \]

\[ \begin{align*}
A &= \frac{w_d}{Q} \\
\gamma &= \frac{1}{m}
\end{align*} \]

\( Q \sim \) oscillations/damping \( \Rightarrow \) light damping

\( \leftrightarrow \) high \( Q \)

\( -m A w_d^2 \cos w_d t - b A w_d \sin w_d t + k A \cos w_d t \neq F_0 \cos w_d t \)

No! The \( \sin \) part is out of phase with the other parts.

\( \Rightarrow \) Improved guess: \( x(t) = A \cos(w_d t - S) \)

Observations \( \Rightarrow A \) is a function of \( w_d \): \( A(w_d) \)

\( A = A_d \) for \( w_d \approx 0 \), \( A \rightarrow \) big near \( w_d = w_0 \), \( A \rightarrow 0 \) for \( w_d \rightarrow \infty \)
Transient behavior
The above solution does not depend on initial conditions → it is only correct after the effect of initial conditions has worn off. One can show that the general solution is
\[ x = A(w_0) \cos[\omega_0 t + \phi(w)] + B e^{-\frac{\alpha}{2} t} \cos(\omega t + \phi) \]

\( \text{steady state} \quad \text{transient} \)
(\( \text{decays away because of } e^{-\frac{\alpha}{2} t} \text{ term} \))

where \( \omega_0 \equiv \omega_0 \), and \( A, B \) are determined by initial conditions.

Waves
An oscillator can be used to create waves that travel, e.g. let the mass splash on a water surface.

Wave types: water, sound, on string, electromagnetic, gravity?!

\( \text{wave moves} \)

Wavenumber \( k = \frac{2\pi}{\lambda} \) \( \text{wavelength} \)