Oscillations
For a mass and spring,
\[ U = \frac{1}{2} k x^2 \]

Any object in stable equilibrium can be modelled as a mass/spring for small displacements.

\[ F_{\text{net}} = F_{\text{spring}} + F_{\text{gravity}} = -kx, \text{ with } x \text{ measured relative to equilibrium.} \]

\[ F = -kx \]
\[ F = m \frac{d^2x}{dt^2} \]

\[ -kx = m \frac{d^2x}{dt^2} \]
A second order differential equation.

The "solution" is the function \( x(t) \) that makes the equation work. There is no general recipe for finding the solutions to 2nd-order DEQs.

One method that sometimes works: any function \( x(t) \) can be expressed in a Taylor series, which we can write \( x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots \)

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Plugging this in to the DEQ sometimes allows one to determine the coefficients.

Let's see if this works in this case:

\[ -k(a_0 + a_1 t + a_2 t^2 + \ldots) = m(2a_2 + 6a_3 t + \ldots) \]

coefficients of like powers of \( t \) on the two sides must be equal

\[ -ka_0 = 2ma_2, \quad -ka_1 = 6ma_3, \ldots \]
(You continue working on this on the next assignment.)

A quicker way of solving DEQs (when it works):

- Guess a solution, based on physical & mathematical intuition
- Plug it in to the DEQ to see if it works.

Observation suggests: \( x(t) \cong A \cos(\omega t + \alpha) \)

Plug this into \( -kx = m \frac{d^2x}{dt^2} \Rightarrow \) It works, if

\[ \omega = \sqrt{\frac{k}{m}} \]

Pendulum:
\[ \omega_{\text{pendulum}} = \sqrt{\frac{g}{L}} \Rightarrow k_{\text{eff}} = \frac{mg}{L} \]

IMPORTANT: \( \omega \) doesn't depend on \( A \)!

\( A, \alpha \) are determined by initial conditions.

Energy \( K + U = \text{const (no air resistance)} \)

\[ E_{\text{tot}} = U_{\text{max}} = \frac{1}{2} k A^2 \]