1. a. Compare the gravitational force felt by NASA’s space shuttle when sitting on the launchpad to the gravitational force it feels when orbiting 350 km above the Earth’s surface. (Earth’s radius is ~6400 km). Are astronauts “weightless” because there is no gravity in space?

\[ F_g = G \frac{m_1 m_2}{d^2} \]

In this problem, \( d \) is the distance between the Earth’s center and the space shuttle, \( m_1 \) is the Earth’s mass and \( m_2 \) is the space shuttle’s mass. “Comparing” the gravitational forces means that we should calculate the ratio of the gravitational forces felt when the space shuttle is on the ground and when the space shuttle is in orbit. The force of gravity is inverse proportional to the distance squared. All other quantities are held constant. Therefore:

\[
\frac{F_{\text{orbit}}}{F_{\text{ground}}} = \frac{d_{\text{ground}}^2}{d_{\text{orbit}}^2} = \frac{6400 \text{ km}^2}{6750 \text{ km}^2} \approx 0.9
\]

The forces are nearly the same. There is gravity in space. Astronauts are not literally “weightless”.

1. b. The asteroid Ida lives in the asteroid belt of our Solar System and is orbited by a tiny asteroidal moon, Dactyl. The average distance between these two bodies is 108 km. The mass of Dactyl is far smaller than that of Ida. What is the orbital period of these two bodies, in hours? The mass of Ida is \( 4.2 \times 10^{16} \) kg.

We need to use the full version of Kepler’s 3rd law, rather than only the proportionality \( P^2 \propto a^3 \) because we don’t have data on other bodies orbiting Ida with which to construct a proportionality.

Therefore:

\[
P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3
\]

\( G = 6.67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \) The mass of Dactyl is much smaller than that of Ida, so we will only use the mass of Ida in the denominator. When plugging in the numbers, we need to convert \( a \) from km into meters by multiplying by 1000 m/km, and we need to be careful to take the square root of both sides. This yields \( 1.3 \times 10^5 \) seconds, or 37 hours.
2. a. What percent of a Sun-like star’s light would an Earth-like planet block out for a distant observer?

As derived in class, the fraction of a star’s light blocked out by a planet is:

\[ f = \frac{R_{\text{planet}}^2}{R_{\text{star}}^2} \]

The radius of the Earth is \(~6400\) km, as given in Problem 1a. The radius of the Sun is \(~700,000\) km. The ratio is thus \(~0.00008\), which means that \(~0.008\)% of the Sun’s light is blocked out by an Earth.

2. b. Consider an M-dwarf star with a radius one-half of the Sun’s radius, a surface temperature that is 4,000 K, and a distance of 750 pc away from us. What percent of this star’s light would an Earth-like planet block out from our point of view?

The only relevant piece of information here is the difference in star radius. The fraction of light blocked out if inversely proportional to the square of the star’s size. Therefore, if the star has \(1/2\) the radius as the Sun, the Earth would block out \(4\) times the light of an M star than it would block out from a Sun-like star.

3. a. Why are the semi-major orbital radii of these first 77 confirmed Kepler planets typically smaller than the orbital radius of the Earth (1 AU)?

This could be a result of true planetary demographics (planets tend to be closer than the Sun than the Earth is) or a result of observational bias. We are biased towards finding planets nearby their parent stars in transit searches, because planets with smaller orbital radii have shorter periods. We need to observe multiple transits to confirm the presence of a planet via this technique. If a planet is very far away from its star, its period will be longer than a year so observing more than one transit could not have happened since 2009.

3. b. Kepler 22b was announced in December 2011 as the first planet discovered in the habitable zone of a Sun-like star. Using data from the table on the Kepler mission website, what is the force of gravity you would feel on planet Kepler 22b compared to the force of gravity that you feel on Earth? Solve as a ratio - don’t plug in actual values.

\[ F_g = \frac{G m_1 m_2}{d^2} \]

in this problem, \(d\) is the radius of the planet, \(m_1\) is the planet’s mass and \(m_2\) is your mass. In one case, the planet is Earth and in another it is Kepler 22b. From the table on the Kepler website, the radius of Kepler 22b is \(~2.4\) Earth radii and its mass is less than \(36\) Earth masses. Let’s just use \(36\) Earth masses for Kepler 22b.

To compare the gravitational force felt by us on Kepler 22b to that felt by us on Earth, we take a ratio. \(G\) and the mass of us will cancel out in the ratio, leaving us with:
3. c. The equilibrium temperature of Kepler 22b is given in the table (assuming a 30% albedo). Calculate this number.

\[ \frac{F_{\text{Kepler}22b}}{F_{\text{Earth}}} = \frac{m_{\text{Kepler}22b}}{m_{\text{Earth}}} \left( \frac{R_{\text{Kepler}22b}}{R_{\text{Earth}}} \right)^2 = \frac{36}{2.4^2} \approx 6.25 \]

\[ T_{\text{planet}} = T_{\text{star}} (1 - A)^{1/4} \left( \frac{R_{\text{star}}}{2d_{s-p}} \right)^{1/2} \]

From the table on the Kepler website, the semi-major axis of the planet’s orbit is ~0.85 AU (which we can consider to roughly equal the average distance between the star and planet), the temperature of the star is 5518 K (similar to the Sun) and the star’s radius is ~0.98 of the Sun’s radius. A = 0.3, as given by the problem.

This problem can be solved either by plugging in all of the numbers, and converting between AU and solar radii, or by calculating it as a ratio compared to the Earth-Sun system. I plugged in. 1 Solar radius ~ 0.004 AU. When using that value, I found 242 K for my equilibrium temperature (compared with 261 K in the Kepler table). If I use 1 Solar radius ~ 0.00465 (more accurate) then I get 261 K.

3. d. How would the equilibrium temperature of Kepler 22b change if its radius and mass both doubled, but everything else about the system remained the same?

The equilibrium temperature of the planet does not depend on its radius or mass, so its temperature wouldn’t change.

4. a. Briefly describe one thing you learned or noticed from looking at the exoplanets.org website.

Full credit for any answer given with care.

b. The figure below shows the planet mass versus its semi-major orbital axis for planets discovered with the velocity wobble technique. There is a relative underdensity of known planets in the upper left (high mass planet near its parent star) and lower right (low mass planet far from its parent star) corners of this figure. For each corner - Does the lack of known planets with those properties necessarily reflect something about the true properties of planetary systems? Briefly explain your answer.

High mass planets with small semi-major orbital axes are the easiest type of exoplanet system to discover with the velocity wobble technique. Therefore, the lack of systems in that region of the figure shows that very massive planets closer than 0.1 AU from their parent star truly are rare. However, low mass planets and planets with large semi-major axis are more difficult to detect with the velocity wobble technique. Therefore, the lack
of low mass planets at relatively larger distances is likely just reflecting an observational bias inherent to this technique.

An aside - One benefit of the transit method: Its not biased for/against planets based on their distance to their parent star (in theory... an experiment needs to be long enough to observe multiple transits). And although it is biased against finding physically small planets - The Kepler mission is sensitive enough to detect Earth’s.