Solutions to Homework 1, Fall 2012

1.

a. From the wikipedia entry for Atom: helium nucleus ~ 1 fm in diameter (10^{-15} meters), helium atom ~ 62 pm in diameter (6.2 * 10^{-11} meters, or ~ 10^{-10} meters)

b. Scale model = 8 inches per nucleus. In this scaled model, the entire diameter of the Helium atom would thus be:

\[ 10^{-10} \text{ meters} \times \frac{8\text{ inches}}{10^{-15} \text{ meters}} = 800,000\text{ inches} \times \frac{\text{foot}}{12\text{ inches}} \times \frac{\text{miles}}{5280\text{ feet}} = 13\text{ miles} \]

The radius of the scaled Helium nucleus would thus be 6.5 miles... placing the edge of the atom somewhere between Wayne and Devon.

c. The fraction of an atom’s volume occupied by the nucleus is:

\[
\frac{V_{\text{nucleus}}}{V_{\text{total}}} = \frac{\frac{4}{3} \pi R_{\text{nuc}}^3}{\frac{4}{3} \pi R_{\text{atom}}^3} = \frac{R_{\text{nuc}}^3}{R_{\text{atom}}^3} = \frac{(10^{-15} \text{ m})^3}{(10^{-10} \text{ m})^3} = \frac{10^{-45} \text{ m}^3}{10^{-30} \text{ m}^3} = 10^{-15}
\]

2. Scaled model of time: 1 year of scaled time = 13.7 billion years of real universe time

a. Telescope was discovered in 1609, or 2012 - 1609 = 403 real years ago.

403 real years * 1 scaled year/13.7 billion real years = 2.94161*10^{-8} scaled years ago

There are about 3.15 * 10^7 seconds per year, so this corresponds to about 0.9 scaled seconds.

The telescope was discovered at 0.0025 seconds before midnight.

b. 200,000 real years * 1 scaled year/13.7 billion real years ~ 1.5*10^{-5} scaled years ago ~ 460 scaled seconds ago ~ 7 minutes ago.

Homo sapiens evolved about 7 minutes ago in our scaled year.
c. The Sun’s formation was 4.6 billion real years ago.

4.6 billion real years * 1 scaled year / 13.7 billion real years ~ 0.336 scaled years ago ~ 4 scaled months ago.

The Sun formed near the beginning of September in our scaled model of the Solar System.

3. While Kepler’s laws were derived empirically from Brahe’s observations of our own Solar System, the same laws would have been derived in any Solar System. One reason we know this is that Kepler’s Laws can be derived from Newton’s universal laws of gravity and motion. Newton’s laws of gravity are fundamental physical laws that apply equally throughout the universe.

4. To find the relative orbital period of Mercury and the (non-existent) planet Vulcan, we rely on Kepler’s 3rd law.

\[ P^2 \propto a^3 \]

We then remember that calculating “relative” quantities requires the calculation of a ratio:

\[ \frac{P_{\text{Vulcan}}}{P_{\text{Mercury}}} = \frac{1}{8} \]

When using Kepler’s third law to calculate this ratio, we remember that taking a ratio of proportional quantities yields us an exact quantity, because the constants of proportionality cancel out in the ratio:

\[ \frac{P_{\text{Vulcan}}^2}{P_{\text{Mercury}}^2} = \frac{a_{\text{Vulcan}}^3}{a_{\text{Mercury}}^3} = \left( \frac{1}{4} a_{\text{Mercury}} \right)^3 = \frac{1}{4^3} = \frac{1}{64} \]

taking the square root of both sides gives:

\[ \frac{P_{\text{Vulcan}}}{P_{\text{Mercury}}} = \frac{1}{8} \]