Bargaining on Appellate Courts

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Abstract

We present a sequential bargaining model of a multi-member appellate court that extends similar models developed to study legislatures. The judges decide a case by majority rule, and announce a legal rule compatible with the case decision. The members of the dispositional majority bargain over which rule to announce. Unique to U.S. Courts, the announced policy must have the support of a majority of the bench, which often requires a super-majority of those in the dispositional majority coalition. We study the properties of equilibria, including the extent of strategic voting, as a function of case location and salience. We show that the median judge is pivotal over case dispositions, but not the announced policy. As delay becomes costless, the dispositional majority endogenously separates into two cohesive factions. The announced policy is the either the ideal policy of some pivotal judge (not necessarily the median), or the result of asymmetric Nash Bargaining between representative leaders of the factions, with bargaining weights proportional to factional size.

Key Words: Bargaining, Judicial Politics,
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1 Introduction

American appellate courts employ four procedures that distinguish them from other multi-member political institutions such as legislatures, juries, administrative agencies, or committees. First and foremost, appellate courts jointly produce a case disposition and a rule. Judges both render judgment, disposing of the case before them, and give reasons for their judgment in an opinion that typically announces a rule or policy. Second, and as a consequence of the first, the members of appellate courts employ a unique compound voting rule to jointly determine the case disposition and policy content of the majority opinion. In particular, majority rule with “universal suffrage” determines the disposition of the case. But a radically different rule governs rule-making: only those judges in the dispositional majority participate in determining the policy the Court will announce in the majority opinion. Moreover, a definitive policy requires the assent of a majority of the court. Consequently, when the dispositional vote is non-unanimous, the voting rule on policy is a super-majority rule, indeed, in some cases, a unanimity rule.

Third, although a particular member of the dispositional majority is tasked with authoring an initial opinion articulating a rule justifying the disposition, his or her opinion may face considerable competition or demands from other members of the dispositional majority. This bargaining is quite unstructured. Fourth, the initial division over the case disposition may reflect strategic voting. Strategic dispositional voting attempts to alter the subsequent bargaining over the opinion. In other words, justices may vote insincerely on the case disposition.

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1Some independent regulatory agencies employ procedures modeled on those of the U.S. Supreme Court. More generally, appellate procedures differ dramatically across legal systems. English courts, for example, follow a *seriatim* practice in which each judge announces her own dispositional vote and reasons for it. The French Cour de Cassation, by contrast, uses a *per curiam* practice in which the court announces a single, unsigned, ostensibly unanimous opinion. For a more general discussion, see Kornhauser (2008).

2Much of the prior literature focuses exclusively on either the disposition, as in Fischman (2008) and Fischman (2011) or Iaryczower and Shum (2012), or on policy as in Spiller and Spitzer (1995), Hammond, Bonneau and Sheehan (2005), Bonneau et al. (2007), and Jacobi (2009). In Lax (2007), courts make policy case by case so dispositions have primacy though the paper does consider the relation between the court’s emergent policy and the announced rules of each judge as well as a game in which judges announce rules rather than policies. Carrubba et al. (2012) and Cameron and Kornhauser (2008) model appellate decision making as a joint product.

3The models in both Carrubba et al. (2012) and Cameron and Kornhauser (2008) recognize that the electorate over the disposition may differ from the electorate over policy. Neither, however, requires that a majority of the court endorse an opinion; indeed, in each model, no such majority opinion will exist for a broad class of parameter values.

4For a description, see e.g., Epstein and Knight. Prior models of the bargaining process either permit only limited opinion competition, as in Lax and Cameron (2007) (where only two opinions compete) or Cameron and Kornhauser (2008) (a monopoly author model), or they do not model opinion competition explicitly, as in Carrubba et al. (2012).
disposition in order to change the identity of the opinion bargainers and thus the content of the majority opinion. Whether they do so presumably depends on the costs and benefits of sincere versus strategic voting.\(^5\)

In this paper, we present a model of multi-member decision-making that incorporates all four of these distinctive features. Thus the model analyzes an appellate court, not a re-labeled model of a legislature (which announces a policy but does not dispose of a case) nor a re-labeled model of a jury (which disposes of a case but does not announce a policy). In order to incorporate the disposition-rule dichotomy, the model is set in case space (Kornhauser (1992a), Kornhauser (1992b), and Lax (2011)) so case dispositions follow logically from the application of general rules to specific cases. The model has two broad stages that correspond to the two decisions required of the Court. The initial stage consists of a vote on the case disposition; the next stage consists of a non-cooperative game-theoretic model of bargaining within the dispositional majority over the content of a majority opinion consistent with the majority’s disposition. Hence, the model distinguishes voting on the case disposition from voting (via so-called joins and concurs on the US Supreme Court) on a policy to be announced in a majority opinion. The outcome of the first stage determines the identity of the second-stage bargainers and places bounds on the possible policy decision, and strategic dispositional voting may further link the two parts of the game.

The model adapts the sequential bargaining approach developed by Baron and Ferejohn and others to study legislatures (Baron and Ferejohn (1989), Banks and Duggan (2006), Jackson and Moselle (2002), McCarty (2000), Kalandrakis (2010), inter alia) to the study of appellate courts. We know of no prior application of sequential bargaining models to institutions that use the peculiar voting rule that U.S. appellate courts commonly use: pure majority rule over the disposition followed by \(k\)-majority rule over policy with an electorate restricted to the members of the dispositional majority. We follow Baron and Ferejohn (1989) and subsequent papers and focus on stationary sub-game perfect equilibria. These equilibria are typically unique and, arguably, have a focal quality. We are able to completely characterize stationary sub-game perfect policy offers and policy voting strategies for all ideological make-ups of all Courts and all discount rates, conditional on the majority dispositional coalition.

As in Rubinstein (1982), Binmore (1987), and Baron and Ferejohn (1989), an important parameter is \(\delta \in [0, 1]\), typically interpreted as the bargainers’ impatience during bargaining.\(^5\)

\(^5\)Cameron and Kornhauser (2008) permit strategic dispositional votes while Carrubba et al. (2012) focus attention on situations in which each judge casts a dispositionally sincere vote. Fischman (2008) and Fischman (2011) also permit strategic dispositional votes, however as noted above judges do not make policy in this model.
(Alternatively, it can be thought of as the cost of making counter-proposals.) In the judicial context, we think $\delta$ is better understood as a toughness parameter that measures the parties’ willingness to compromise. High levels of $\delta$ lead to “tough” bargaining in which non-writers demand compromises from the opinion author if they are to join the majority opinion. Low levels of $\delta$ lead to softer bargaining in which non-writers are willing to join opinions with few compromises. Plausibly, $\delta$ rises with a case’s importance, and with greater resources per case. Thus, changes in $\delta$ are readily interpretable as changes in substantively important attributes of cases such as their importance, or attributes of different courts such as case load and resources. In this paper, we pay particular attention to the behavior of judges as $\delta \to 1$.

The model makes three substantive contributions to our understanding of appellate adjudication, as well as offering a novel application of the sequential bargaining approach to political institutions.

First, the model analyzes the impact of strategic dispositional voting on opinion content and case dispositions. Because strategic dispositional voting appears to be common in some appellate courts such as the U.S. Courts of Appeals, this analysis is arguably important. In the model, strategic dispositional voting can be quite consequential for opinion content. However, when the location of the case before the court lies within the interval spanned by the judges’ ideal points, strategic dispositional voting does not alter the winning disposition. On the other hand, if the case is extreme (but not too extreme) and $\delta$ is sufficiently small, strategic dispositional voting may actually change the disposition of the case. Under strategic dispositional voting, unanimous dispositions are the predominant outcome (as they are on the U.S. Courts of Appeals).

Second, the toughness or case importance $\delta$ strongly affects the character of the bargaining. Although agreement is immediate, many values of $\delta$ lead ex ante to a distribution of opinion locations rather than a single location. The distribution reflects probabilistic opinion assignment during the bargaining game over policy and the fact that values of $\delta$ even modestly lower than 1 lead to a pronounced first-mover advantage for the realized assignee. Indeed, for sufficiently low values of $\delta$, the assignee is able to impose her most-preferred policy on the majority coalition. However, as $\delta$ goes to 1, the distribution of equilibrium opinion locations converges to a single point, so the first-mover advantage degrades.

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Using a data set of immigration decisions concerning asylum and a theoretical model in which judges on a three-judge panel decide only the disposition, Fischman (2008) estimates that at least one judge votes strategically in 45% of the cases and two judges vote strategically in 8% of the cases.
We provide an explicit characterization of the limit equilibria, extending insights presented in Predtetchinski (2011). We verify the well known result, that under simple majority rule (which in our model translates to the case of a unanimous dispositional vote), the limit policy coincides with the median judge's ideal (see Cho and Duggan (2009)). However, when the decision is non-unanimous, the limit policy neither privileges the median judge on the bench (as predicted by Hammond, Bonneau and Sheehan (2005)) nor the median judge within the majority coalition (as predicted by Cardona and Ponsatí (2007)). Nor need it coincide with some 'average' policy. Instead, the limit policy coincides with the asymmetric bilateral Nash Bargaining solution between two ‘decisive’ judges in the dispositional majority. The bargaining weights are determined endogenously in equilibrium, as a function of the ideal policies of all agents in the dispositional majority. We show that the limit policy is generically a strict subset of the ‘core’. Moreover, depending on the arrangement of judges’ ideal policies, the limit policies may coincide with the ideal policies of a subset of judges in the dispositional majority (not necessarily the median).

Third, the location of the selected case affects both the disposition of the case and the content of the majority opinion under both sincere and strategic voting. Hence, the model suggests that case selection matters in appellate jurisprudence because some cases conduce to particular policy outcomes rather than others. Again, a full discussion is beyond the scope of this paper, but a ”bargaining vehicle” theory of case selection is quite different from current theories of certiorari highlighting control or learning in the judicial hierarchy (Cameron, Segal and Songer (2000), Carrubba and Clark (2012), Beim (2017)).

The paper is organized in the following way. Section II presents the model. Section III examines policy bargaining within the dispositional majority. For clarity of exposition, we focus on bargaining majorities created by sincere dispositional voting. Section IV analyzes dispositional voting. Section V concludes. All proofs are presented in the Appendix.

2 The Model

2.1 Cases, Dispositions and Rules

There is a court consisting of \( n \) judges (assumed odd), which must decide a case. A case \( z \) encodes the details of an event that has occurred, for example, the level of care exercised by a manufacturer or the intrusiveness of a search by the policy. Let \( Z = [0, 1] \) be the case
space. A judicial disposition \( d \in D = \{0, 1\} \) of the case determines which party prevails in the dispute between the litigants. Judges dispose of cases by applying a legal rule.

A legal rule \( r : Z \to D \) maps the set of possible cases into dispositions. Let \( X = 2^Z \) be the space of possible rules. We focus on an important class of legal rules, cutpoint-based doctrines, which take the form:

\[
r(z; y) = \begin{cases} 
1 & \text{if } z \geq y \\
0 & \text{otherwise}
\end{cases}
\]

where \( y \) denotes the cutpoint. For example, in the context of negligence, the defendant is not liable if she exercised at least as much care as the cutpoint \( y \).\(^7\) Let \( X^C \) be the space of cut-point rules. We have \( X^C = \{[[0, y), [y, 1]] \mid y \in [0, 1]\} \). It should be clear that rules live in an entirely different space to cases. The special structure of cut-point rules allows us to summarize them in terms of a threshold in case-space.

### 2.2 Decision Making by the Court

The task of the justices is twofold: first, to dispose of the case in hand using pure majority rule, and second, to propose and pick a legal rule using a \( k \)-majority rule within the dispositional majority coalition. A policy is a proposal (a “draft opinion”) from a judge in the majority dispositional coalition; a successful policy proposal thus attracts an endorsing vote (a “join” versus a “concur”) from at least \( k \) justices in the majority dispositional coalition. The case disposition and the rule must be consistent in the sense that the applying the rule to the case gives the chosen disposition. For example, following a case \( z \), if the majority chooses disposition 1, then the chosen rule \( y \) must satisfy \( y \leq z \).

Following the procedure of the U.S. Supreme Court, \( k \) corresponds to a bare majority of the entire Court regardless of the number of justices in the majority dispositional coalition. So, \( k = 5 \) on a 9-member court like the U.S. Supreme Court while \( k = 2 \) on a 3-judge court like a U.S. Court of Appeals. Because the dispositional majority may vary in size from a bare majority of the Court to its entire membership, the \( k \)-majority rule for policy-making may

\(^7\)Other examples include allowable state restrictions on the provision of abortion services by medical set providers; state due process requirements for death sentences in capital crimes; the degree of procedural irregularities allowable during elections; the required degree of compactness in state electoral districts; and the allowable degree of intrusiveness of police searches. Many other examples of cutpoint rules may suggest themselves to the reader.
effectively be a simple majority rule, a super-majority rule, or a unanimity rule, depending on the size of the dispositional majority.

The justices in the dispositional majority bargain over the rule to be implemented. We formalize this by studying a bargaining model à la Baron and Ferejohn (1989) and Banks and Duggan (2000). A judge from the dispositional majority is recognized to propose a policy $y$ that is consistent with the majority’s disposition. Upon seeing the proposal, each justice in the dispositional majority either votes to endorse the proposed opinion by ‘joining’ or declines to endorse the opinion by ‘concurring’. If at least $k$ justices join the opinion, then it becomes the policy of the court. Else, the justices retire, and the process repeats itself in the following period, and this continues until a policy of the court emerges. In the first period of bargaining, we allow the identity of the proposing judge to be non-random, reflecting current practice where the most senior judge in the dispositional majority determines who will write the opinion. However, in subsequent bargaining periods, we assume judges are recognized with equal probability, reflecting the equal right of judges to make counter-proposals.

### 2.3 Judicial Preferences

Following Carrubba et al. (2012) and Cameron and Kornhauser (2008), we distinguish the utility received from casting dispositional votes from the utility derived from the content of majority opinions.\(^8\)

Suppose judge $j$ has ideal threshold $x^j$, and that $0 \leq x^1 \leq \ldots \leq x^n \leq 1$, so that the judges are ordered by ideal threshold. Judge $j$’s policy utility from generic rule $y$ is:

$$u_P(y; x^j) = 1 - (y - x^j)^2$$

where $u(z)$ is concave and achieves a maximum at $z = 0$. In effect, we assume that preferences satisfy the ‘symmetry’ assumption in Cardona and Ponsati (2011), which is tantamount to assuming that preferences are identical up to a translation. Notice that $u(y; x^j) \geq 0$ whenever $y \in [0, 1]$. We normalize the disagreement payoff to 0.

Judge $j$’s dispositional utility is:

$$u_D(d^j; z, x^j) = \begin{cases} 0 & d^j = r(z; x^j) \\ -D|z - x^j| & d^j \neq r(z; x^j) \end{cases}$$

\(^8\)Cameron and Kornhauser (2008) treats the utility of casting join vs concur votes as expressive; in contrast, here the value of such votes comes from the policy resulting from votes.
where \( d_j \) is judge \( j \)’s dispositional choice during the first stage of decision making, and \( D \geq 0 \).

The judge incurs a penalty if she casts an insincere or ‘strategic’ dispositional vote, reflecting her personal desire to try to affect the ‘correct’ outcome, by her lights. Note that, in contrast to policy utility, where the judge’s utility depends on the policy of the court, dispositional utility stems purely from the judge’s individual choice. Hence, switching from one dispositional vote to another necessarily brings a change in utility even if one’s dispositional vote is not pivotal in determining the Court’s disposition of the case.

We should note the role of the “legal status quo” within the bargaining game, as this point has engendered some controversy among judicial scholars. Although there is a prior legal policy, this policy effectively reverts to a null policy when the Court takes the case – policy is in limbo until the Court resolves the case. In fact, the only way for policy to revert to the status quo \textit{ante} is for the Court to re-enact it anew in the majority opinion. So, the players do not receive a ‘status quo’ payoff during rounds of bargaining, but rather zero each round there is no agreement on a new policy. We finesse the issue of whether the Court would ever take another case in a policy area, that is, whether the announced policy is actually time-consistent and renegotiation-proof. In effect, we assume the Court commits to its announced policy, but we do not study the commitment mechanism. None of the current models of collegial courts address the time-consistency of the bargaining outcome (but note Rasmusen (1994) and Cameron, Kornhauser and Parameswaran (2017)). Recent legislative models of sequential policy making with evolving status quos determined by earlier rounds of policy-making are suggestive (Baron and Ferejohn (1989), Kalandrakis (2010)) but we do not pursue this point any farther in this paper.

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\footnote{The choice of functional forms for policy and dispositional utilities are not arbitrary, and satisfy an internal consistency that is explored in greater detail in Cameron, Kornhauser and Parameswaran (2017). In brief, suppose judge \( j \)’s primitive preferences are only over dispositions, and that she receives utility 0 if cases are decided correctly, and \(-|z-x|\) if case \( z \) is decided incorrectly. (Such preferences satisfy the \textit{increasing differences in dispositions} property that that paper explores.) If cases are drawn from a uniform distribution \( z \sim U[E[z]-\varepsilon, E[z]+\varepsilon] \), it can be easily shown that the \textit{ex ante} dispositional utility from applying rule \( y \) is \(-\frac{1}{1-p} (y-x)^2\). Suppose the court decides in the instant case to apply rule \( y \) in the current and all future cases, and suppose judges discount the future at rate \( \rho \). Then, if rule \( y \) correctly decides the instant case, judge \( j \)’s expected lifetime utility is \(-\frac{1}{1-p} \frac{1}{4\varepsilon} (y-x)^2\). By contrast, if rule \( y \) incorrectly decides the instant case, her expected lifetime utility is \(-|z-x| - \frac{\rho}{1-p} \frac{1}{4\varepsilon} (y-x)^2\). These are identical to our current preferences if we set \( u(z) = -\frac{1}{4\varepsilon} \frac{1}{1-p} z^2 \), noting that utilities are unique up to a positive affine transformation. Moreover, applying the appropriate transformation gives \( D = 4\varepsilon \frac{1-p}{\rho} \), so that \( D \) becomes larger as the judge becomes more impatient or when the case generating process is more dispersed.}
2.4 Strategies and Continuation Values

Given the repeated game structure of bargaining, strategies can be quite complex, as they may be history dependent. We restrict attention to stationary strategies, which require the players to choose equivalent strategies in every structurally equivalent sub-game.

A strategy for judge $j$ is a triple $(d^j, y^j, A^j)$, where:

- $d^j(z; x, \delta)$ denotes the judge’s dispositional vote, given a case $z$, the vector of ideal policies $x = (x_1, \ldots, x_n)$, and the discount factor $\delta$.
- $y^j(z; x, M, \delta)$ denotes the policy proposed by the judge, whenever she is in the dispositional majority and recognized to make a proposal, where $M \subseteq \{1, \ldots, n\}$ is the composition of the dispositional majority.
- $A^j(z; x, M, \delta)$ denotes the set of proposals that the judge will accept, whenever she is in the dispositional majority.

We solve for sub-game perfect equilibria. We restrict attention to no-delay equilibria, which are well known to exist in bargaining games of this sort (see Banks and Duggan (2006)).

3 Bargaining over Policies within the Dispositional Majority

3.1 Equilibrium Characterization

Let $z$ be the case, and suppose the dispositional majority coalition $M \subseteq \{1, \ldots, n\}$ contains $m \in \{k, \ldots, n\}$ members, where $k = \frac{n+1}{2}$. Without confusion, we re-label the judges in the coalition, preserving the ordering of ideal policies, so that $M = \{1, \ldots, m\}$ with $x_1 \leq \ldots \leq x_m$.

(Once the majority coalition has been determined, the preferences of the non-majority judges become inconsequential, so we are free to disregard them, and focus on the $m$ remaining judges.)

If the majority disposition was 1, the majority must choose a policy in the interval $[0, z]$, whilst if the disposition was 0, it must choose a policy in the interval $[z, 1]$. Generically, the court’s policy must be contained in $[\underline{y}, \bar{y}]$, where $\underline{y} \in \{0, z\}$ and $\bar{y} \in \{z, 1\}$. 
Let $y^i \in [\underline{y}, \bar{y}]$ be the equilibrium proposal of judge $i$ in the majority. Let $E[y] = \frac{1}{m} \sum_i y^i$ be the expected policy, and let $Var(y) = \frac{1}{m} \sum_i (y^i - E[y])^2$ be the variance in equilibrium policies. Since we study no-delay equilibria, in any sub-game, the judges should expect the offer in the subsequent bargaining round to be accepted. Hence, the payoff to judge $j$ in the continuation play is:

$$v^j = \frac{1}{m} \sum_{i=1}^{m} \left[ 1 - \left( y^i - x^j \right)^2 \right] = 1 - \left( E[y] - x^j \right)^2 - Var(y)$$

Let $C^j \subseteq M$ denote the coalition that supports judge $j$’s proposal in equilibrium, possibly including $j$ herself. Since a proposal requires at least $k$ joins to be implemented, $\#C^j \geq k$ for all $j$. Note that $\{1, ..., k\}$ and $\{m - k + 1, ..., m\}$ are both winning coalitions. Before characterizing the equilibrium of the bargaining game, we first note the following Lemma regarding the composition of the equilibrium coalitions:

**Lemma 1.** The equilibrium coalition for judge $j$ is connected, and satisfies:

- $\{1, ..., k\} \subseteq C^j$ if $x^j \leq E[y]$
- $\{m - k + 1, ..., m\} \subseteq C^j$ if $x^j > E[y]$.

Suppose judge $j$’s ideal rule is lower than the expected rule that will be chosen in the continuation play. Lemma 1 states that, in equilibrium, judge $j$ will propose a policy $y^j$ that is joined by at least the $k$ judges with the lowest ideal policies. To understand why, first note that proposing $y' = E[y]$ would earn the support of all $m$ judges in the coalition. (This is true for two reasons. First, the strict concavity of policy preferences implies that every judge would prefer to receive the expected policy with certainty than to face a lottery over policies with the same expectation. Since judges face exactly such a lottery in the continuation game, each would strictly prefer to receive the expected policy for sure in the current period. This would be true, even if delay were costless. But second, delay is costly, which makes the compromise even more preferred.) Next, starting from $E[y]$ as judge $j$ pulls the proposed policy towards her ideal $x^j$, she will progressively lose the support of judges. Moreover, the first judge to leave the coalition will be judge $m$ (with the highest ideal policy), then judge $m - 1$ etc. Hence, for any proposal, the coalition that supports it will be connected, and include all the judges, except perhaps the upper-most subset. Since in equilibrium, the policy must have the support of at least $k$ judges, it must have the support of judges $1, ..., k$. 

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Judge $j$ will continue to pull the policy towards her ideal, either until she reaches her ideal, or she loses the support of judge $k$. A similar intuition holds for the case of $x^i > E[y]$.

We are now ready to characterize the equilibrium of the bargaining game.

**Proposition 1.** Consider a bargaining game with ideal policies $\{x^1, ..., x^m\}$ and baseline case $z$. For $\delta < 1$, the game admits a unique equilibrium, characterized by a pair $(y^L, y^H)$, with $\underline{y} \leq y^L \leq y^H \leq \bar{y}$, such that:

1. When judge $j$ is recognized, she will propose: $y^j = \begin{cases} y^L & x^j < y^L \\ x^j & x^j \in [y^L, y^H] \\ y^H & x^j > y^H \end{cases}$

2. The pair $(y^L, y^R)$ satisfies:
   - $y^L = \max \left\{ \underline{y}, x^k - \sqrt{(1 - \delta) + \delta (E[y] - x^k)^2 + \delta \text{Var}(y)} \right\}$
   - $y^H = \min \left\{ \bar{y}, x^{m-k+1} + \sqrt{(1 - \delta) + \delta (E[y] - x^{m-k+1})^2 + \delta \text{Var}(y)} \right\}$

Moreover, the equilibrium mapping is continuous in $(x, z, \delta)$.

Roughly speaking, Proposition 1 states that judges with ‘moderate’ preferences will be able to successfully implement their ideal rule in equilibrium, whilst judges with (relatively) ‘extreme’ preferences must offer a compromise rule. All ‘extreme left’ judges will pool on the same proposal $y^L$, whilst all ‘extreme right’ judges will pool on the same proposal $y^R$.$^{10}$

What constitutes ‘moderate’ and ‘extreme’ is itself determined in equilibrium. By Lemma 1, we know that any equilibrium policy must have the support of both judge $m - k + 1$ and judge $k$. We refer to these judges as the left and right decisive judges, respectively. By construction (in part 2 of the proposition), $y^L$ is the lowest policy that the right decisive judge ($k$) will accept, and $y^H$ is the highest policy that the left decisive judge ($m - k + 1$) will accept. Any proposal in the region $[y^L, y^H]$ is thus equilibrium consistent.

Proposition 1 does not provide closed form expressions for $y^L$ and $y^H$. Instead, as is common in bargaining models, these are defined recursively. Given a conjecture $(y^L, y^H)$, part

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$^{10}$In fact, Cardona and Ponsati (2011) show that this is a general feature of bargaining equilibria in one dimensional spaces. Cho and Duggan (2003) show that bargaining equilibria are unique when preferences are quadratic.
1 of the proposition determines each judge’s optimal proposal, which in turn determines the mean and variance in offers. The implied values \((\hat{y}_L, \hat{y}_H)\) can then be imputed by part 2 of the proposition. If the conjectured and implied thresholds coincide, they constitute an equilibrium. In fact, as the proof demonstrates, the mapping from conjectured to implied thresholds is a contraction mapping. This implies that the equilibrium is unique. Additionally, the equilibrium can be computed iteratively by starting with any arbitrary conjecture, and updating repeatedly.

### 3.2 Comparative Statics on \(\delta\)

The discount rate \(\delta\) parameterizes the cost of delay in bargaining. Given the bargaining dynamic, this entails that it is also a measure of the degree of agenda control that the proposer exerts. When \(\delta = 0\), delay is so costly that the non-proposing judges will accept any feasible policy. The proposer thus has complete control over the agenda and will propose the feasible policy closest to her ideal.

Before proceeding to the analysis, we remind the reader that the vector of equilibrium proposals \(y\), as well as \(y^L, y^H, E[y]\) and \(Var(y)\) are all functions of \(\delta\) (as well as the vector of ideal policies \(x\)). In this section, we make the dependence on \(\delta\) explicit, by denoting \(y^L_\delta\), \(y^H_\delta\) and so on.

As the following lemma shows, as \(\delta\) increases, the range of proposals that are equilibrium consistent narrows. Moreover, in the limit as \(\delta \to 1\), the proposer loses control of the agenda entirely, and in equilibrium, all judges will make the same proposal.

**Lemma 2.** In any equilibrium, \(y^H_\delta > y^L_\delta\) and \(Var(y_\delta) > 0\) whenever \(\delta < 1\). Moreover, \(y^H_\delta - y^L_\delta\), and \(Var(y_\delta)\) are both decreasing in \(\delta\), and \(\lim_{\delta \to 1} Var(y_\delta) = 0\).

Lemma 2 states the familiar result that when delay is costly (\(\delta < 1\)), the proposer can pull the chosen policy somewhat towards her ideal. This results in equilibrium proposals that are dispersed around a mean. The lemma additionally shows that as \(\delta \to 1\), the proposer’s advantage disappears, and the best the proposer can do is to announce the expected policy. (See Predtetchinski (2011).)

Taking the limit as \(\delta \to 1\) is also interesting, because it allows a fairer comparison between the bargaining model and models that are predicated on median-voter logic (or that predicts outcomes in the ‘core’). The logic behind such models is that, if some
other policy were proposed, a counter-proposal would necessarily exist that had the support of a majority of judges. But, in the bargaining model, even if it existed, such a policy might not be counter-proposed if the cost of delay was sufficiently high (i.e. if $\delta$ is small enough). By contrast, as $\delta \to 1$, the cost of delay becomes arbitrarily small, and so, if a majority-preferred deviation exists, we should expect to see a counter-proposal.

**Lemma 3.** Suppose $\delta = 1$. Then, for any $y \in [x^{m-k+1}, x^k] \cap [\underline{y}, \bar{y}]$, there is an equilibrium in which all judges propose $y^j = y$.

The set $[x^{m-k+1}, x^k] \cap [\underline{y}, \bar{y}]$ represents the ‘core’.\(^\text{11}\) It is the set of feasible proposals for which there is not some other proposal that is preferred by at least $k$ judges. If $m = n$, the core is uniquely the median of the judges (if feasible, or $z$ otherwise). Else, it represents a range of policies. Lemma 3 states that when $\delta = 1$, there are generically multiple equilibria, and that any policy in the core can be sustained as a bargaining equilibrium. This is consistent with standard intuitions from social choice theory.

Although multiple equilibria typically exist at $\delta = 1$, we know that equilibria are unique when $\delta < 1$. Moreover, these equilibria are continuous. Taking the limit of these equilibria as $\delta$ approaches 1, we can isolate a particular ‘focal’ equilibrium at $\delta = 1$ from amongst the many. This focal equilibrium is robust in the sense that it is the only one that continues to survive when bargaining becomes slightly costly.

### 3.3 Limit Equilibria

Before we characterize the limit equilibria, we first consider an alternative two-stage game, which we dub the *faction formation game*. There are two factions, $L$ and $R$, led by the left and right decisive judges, respectively. In the first stage, each judge $i$ (simultaneously) chooses an amount of support $\rho_i \in [0, 1]$ to give to the $R$ faction, with the remainder $(1 - \rho_i)$ going to the $L$ faction. The total support for the factions is $\omega_L = \frac{1}{m} \sum_i (1 - \rho_i)$ and $\omega_R = \frac{1}{m} \sum_i \rho_i$. In the second stage, the policy is determined as a consequence of asymmetric Nash Bargaining between the left and right decisive judges, with bargaining weights equal to the support for each faction. Hence, following a history where factional supports $(\omega_L, \omega_R)$ emerge, the resulting policy $b_\omega$ satisfies:

$$b_\omega = \arg \max_b \left[ 1 - \left( b - x^{m-k+1} \right)^2 \right]^{\omega_L} \left[ 1 - \left( b - x^{k} \right)^2 \right]^{\omega_R}$$

\(^{11}\)The notation is a little sloppy here. It is possible that $[x^{m-k+1}, x^k] \cap [\underline{y}, \bar{y}] = \emptyset$ if $x^{m-k+1} < x^k < z$ and $d = 0$ or if $z < x^{m-k+1} < x^k$ and $d = 1$. In this case, define the core to be $\{z\}$. 

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It should be clear that the second stage is purely mechanical, and that all of the action is in the first stage.

**Proposition 2.** There exists a judge \( i \in \{m - k + 1, ..., k \} \) s.t. in any Nash equilibrium: (i) if \( x^j < x^i \), then \( \rho_j = 0 \), and if \( x^j > x^i \), then \( \rho_j = 1 \), and (ii) \( \rho_i > 0 \), and \( \rho_i < 1 \) only if \( b^\omega = x^i \).

The policy that emerges in the second stage is monotonically increasing in the size of the \( R \) faction’s support; giving support to faction \( R \) causes the equilibrium policy to move closer to the right decisive judge’s ideal policy, *ceteris paribus*, and vice versa. Intuitively, then, most judges will not find it optimal to split their support between both factions, since doing so pulls policy in opposite directions, negating the overall effect. Proposition 2 formalizes this intuition by showing that if some judge splits his support between the factions, then all judges with ideal policies to his left must give all their support to the \( L \) faction, and all judges with ideal policies to his right must give all their support to the \( R \) faction. Moreover, a judge will only split his support between the factions, if the equilibrium policy that results coincides with his ideal policy.

Since judges’ ideal policies are not distinct, there might be several judges who ‘mix’ in equilibrium. We refine our equilibrium by focusing on the subset of equilibria that are monotone and involve mixing by only one judge. We have the following:

**Corollary 1.** There exists an equilibrium characterized by judge \( i^* \in \{m - k + 1, ..., k \} \), s.t. (i) \( \rho_{i^*} > 0 \), and (ii) \( \rho_j = 0 \) for all \( j < i^* \) and \( \rho_j = 1 \) for all \( j > i^* \). The equilibrium policy is \( x^i \) if \( \rho_{i^*} < 1 \), and is the Nash bargaining solution:

\[
b^* = \arg \max_b \left[ 1 - (b - x^m)^2 \right]^{1 - \frac{\omega}{m}} \left[ 1 - (b - x^k)^2 \right]^{\frac{\omega}{m}}
\]

otherwise.

The equilibrium of the faction formation game identifies a decisive player \( i^* \), such that the resulting policy either coincides with \( i^* \)’s ideal policy, or is the result of Nash Bargaining between two endogenous coalitions, where the \( L \) faction is comprised of judges \( 1, ..., i^* - 1 \), and the \( R \) faction is comprised of judges \( i^*, ..., m \). It turns out that the limit equilibrium of the bargaining game of interest coincides with the equilibrium policy that results from the faction formation game. Our model thus provides micro-foundations for the emergence of cohesive policy factions on the Supreme Court.
Before stating the formal result, some notation. Let $\mu = \lim_{\delta \to 1} E[y_\delta]$ and note, by Lemma 2, that $\lim_{\delta \to 1} y^L_\delta = \mu = \lim_{\delta \to 1} y^H_\delta$. For each $j = 1, ..., m$, let

$$b^j = \arg \max_b \left[ 1 - (b - x^{m-k+1})^2 \right]^{1-m} \left[ 1 - (b - x^k)^2 \right]^{1-m}$$

denote the asymmetric Nash Bargaining solution when agents $\{1, ..., j - 1\}$ join faction $L$ and agents $\{j, ..., m\}$ join faction $R$. Note that $b_j$ is decreasing in $j$.

\textbf{Proposition 3.} Let $i^* = \min \{i \mid x^i \geq b^{i+1}\}$. Then:

- If $x^{i^*} > b^{i^*}$, then $\mu = \lim_{\delta \to 1} E[y_\delta] = b^{i^*}$.
- If $x^{i^*} \leq b^{i^*}$, then $\mu = \lim_{\delta \to 1} E[y_\delta] = x^{i^*}$.

Moreover, if $\mu = b^{2\pi+1}$, or if $x^{i^*} = \frac{1}{2} (x^k + x^{m-k+1})$, then there exists $\delta < 1$ s.t. $E[y_\delta] = \mu$ for all $\delta > \delta$. (Caveat: $\mu = z$ if $b^{i^*} \notin [y, \bar{y}]$ or $x^{i^*+1} \notin [y, \bar{y}]$, as appropriate.)

Proposition 3 characterizes the common equilibrium policy that all judges will propose when the cost of making counter-proposals becomes arbitrarily small. Depending on the arrangement of the judges’ ideal policies, the limit proposal may either: (i) coincide with the ideal policy of one of the judges $m - k + 2, ..., k - 1$, or (ii) be the solution to the asymmetric Nash Bargaining problem between the left and right decisive judges $(m - k + 1$ and $k)$, with weights proportional to the support for each faction. To our knowledge, this is the first paper to provide a simple and intuitive characterization limit results for this (common) bargaining game.

To build intuition for this result, conjecture that $\mu \in (x^{i-1}, x^i)$. Our goal is to investigate the conditions under which this will be equilibrium consistent. Since $\text{Var}(y_\delta) \to 0$ as $\delta \to 0$, for $\delta$ large enough, $x^{i-1} < y^L_\delta < E[y_\delta] < y^H_\delta < x^i$. Hence, when $\delta$ is large, judges $1, ..., i - 1$ will propose $y^L_\delta$ and judges $i, ..., m$ will propose $y^H_\delta$. It is as if the judges separate into cohesive factions, with all members of the same faction making the same equilibrium proposal. Note, further, that $y^L_\delta$ and $y^H_\delta$ were determined by the preferences of the right and left decisive judges, respectively. Hence, we can think of the left decisive judge as bargaining on behalf of the left faction, and the right decisive agent as bargaining on behalf of the right faction, with the bargaining weights being proportional to the size of their respective factions. As $\delta \to 1$, this coincides with the asymmetric Nash bargaining solution between the left and right decisive agents (see Binmore, Rubinstein and Wolinsky (1986)).
Notice that the separation into factions was endogenous to the equilibrium. Hence, for the asymmetric Nash bargaining solution to indeed be equilibrium consistent, it must be that this solution lies in the interval \((x_{i-1}, x_i)\) — so that players separate into the factions as conjectured. As the proof of Proposition 3 shows, there is a unique player \(i^*\) that determines the composition of factions, in equilibrium. There are two possibilities. For a range of recognition probabilities and ideal policy arrangements, the factions \(\{1, \ldots, i^* - 1\}\) and \(\{i^*, \ldots, n\}\) induce a faction-size weighted Nash Bargaining solution that is equilibrium consistent — i.e. which falls in the required interval \((x_{i^*-1}, x_{i^*})\). (In the faction formation game, \(\rho_{i^*} = 1\), so that \(i^*\) gives all his support to faction \(R\).) Outside this range of parameters, the following problem arises: If \(i^*\) is conjectured to be in the left faction, then the location of the induced Nash Bargaining solution will cause \(i^*\) to defect to the right faction, and vice versa. Player \(i^*\) is pivotal. (In the faction formation game, \(\rho_{i^*} \in (0, 1)\).) The only possibility is that the limit equilibrium coincides with \(i^*\)'s ideal policy, \(x_{i^*}\).

The following example may help in building further intuition for the results in Lemma 3:

**Example 1.** Suppose \(m = k = 5\), so that \(m - k + 1 = 1\). This coincides with scenario in which the dispositional coalition consists of a bare majority of the Supreme Court. We normalize the ideal policies of the highest and lowest judges to: \(x_1 = 0.2\) and \(x_5 = 0.8\). Then the Nash Bargaining solutions are approximately: \(b_1 \approx 0.8\), \(b_2 \approx 0.6604\), \(b_3 \approx 0.5505\), \(b_4 \approx 0.4495\) and \(b_5 \approx 0.3396\). Then:

\[
\mu = \begin{cases} 
  b_2 \approx 0.6604 & x^2 > 0.6604 \\
  x^2 & 0.5505 \leq x^2 \leq 0.6604 \\
  b_3 \approx 0.5504 & x^2 < 0.5505 < x^3 \\
  x^3 & 0.4495 \leq x^3 \leq 0.5505 \\
  b_4 \approx 0.4495 & x^3 < 0.4495 < x^4 \\
  x^4 & 0.3396 \leq x^4 \leq 0.4495 \\
  b_5 \approx 0.3396 & x^4 < 0.3396 
\end{cases}
\]

First, let us check the logic of the Lemma through the example. Suppose \(x^2 = 0.5 < 0.7 = x^3\). Conjecture that \(\mu \in (x^2, x^3)\). Then, per the logic in the previous paragraph, for \(\delta\) large enough, it must be that judges 1 and 2 choose \(y^L_5\) and judges 3, 4 and 5 choose \(y^H_5\). It follows that the equilibrium policy will be the Nash Bargaining solution when the left decisive voter has weight \(\frac{2}{5}\) and the right decisive voter has weight \(\frac{3}{5}\). This solution is \(b_3 \approx 0.5504\), which lies between \(x^2\) and \(x^3\), as conjectured. By contrast, suppose that \(x^2 = 0.6 < 0.7 = x^3\), and conjecture again that \(\mu \in (x^2, x^3)\). The same logic would lead us to conclude that \(\mu = b_3\). But
this no longer satisfies the requirement that $x^2 < \mu < x^3$. Our conjecture is not equilibrium consistent. We can similarly show that conjecturing $\mu \in (x^1, x^2)$ is not consistent. Under this arrangement of ideal policies, judge 2 is extreme pivotal. Conjecturing that his ideal policy is below the limit expected policy causes us conclude that it is, in fact, above. Instead conjecturing that his ideal policy is above the limit expected policy causes us to conclude that it is below. The only consistent alternative is that the limit policy coincides with $x^2$.

Next, let us note some features of the equilibrium mapping. First, for each judge strictly between judge $m - k + 1$ and judge $k$, there is some arrangement of ideal policies for which they are pivotal. With 5-member majority coalition, it is possible that equilibrium policies reflect the ideal policies of any of judges 2, 3 and 4, as the cost of delay vanishes. In particular, the median judge in the majority is not generically privileged. Additionally, there are arrangements of ideal policies under which no judge is extremely pivotal, and the equilibrium policy is simply (approximated by) a weighted sum of the ideal policies of judge $m - k + 1$ and judge $k$. Note that, although the ideal policies of the remaining judges does not affect the equilibrium policy, these will matter in so far as they determine the weights attributed to $x^k$ and $x^{m-k+1}$.

Second, for different arrangements of ideal policies, the limit equilibrium selects some policy in the core. However, not every policy in the core can be sustained in equilibrium. In particular, policies too close to $x^k$ and $x^{m-k+1}$ are not sustainable. These intuitions hold even if several judges had identical preferences and voted as a block. For example, suppose $x^2 = x^3 = x^4$, so that the middle 3 judges constitute a homogeneous middle bloc. Then this middle bloc will be pivotal provided that $x^2 \in [b^5, b^2]$. For $x^2$ outside this interval, the ‘extreme’ judges will continue to exert some influence over the location of the equilibrium policy. By contrast, suppose $x^3 = x^4 = x^5$, so that there is a right-wing bloc. Notwithstanding that this bloc contains the median justice in the majority, it will never be pivotal, and the equilibrium policy will never reflect the bloc’s ideal. Moreover, if $x^2 \in (b^5, b^2)$, then judge 2 will be pivotal.

Finally, the last line of Proposition 3 deserves mentioning. For the most part, the results in Proposition 3 are limit results — they describe the location of $E[y_{\delta}]$ as $\delta \to 1$. However, if the arrangement of ideal policies is symmetric, in the sense that the number of judges with ideal policies lying on either side of this limit policy is equal, then the result holds not just in the limit, but for all $\delta$ sufficiently large. The intuition is that, with this symmetry, $y_{\delta}^L$ and $y_{\delta}^H$ will be evenly spaced around $\mu$. (Since $\mu$ is exactly at the midpoint of $x^k$ and $x^{m-k+1}$,
judges $k$ and $m - k + 1$ will be equally willing to accommodate compromise policies, and so $E [y_\delta] - y_\delta^L = y_\delta^H - E [y_\delta].$ But then, since an equal number of judges propose $y_\delta^L$ and $y_\delta^H$, the implied expected policy $E [y_\delta]$ will coincide with $\mu$. Two special cases are highlighted in the following Corollary:

**Corollary 2.** The following are an immediate consequence of Proposition 3:

- Suppose $m = n$, so that the majority coalition contains the entire bench. Then, there exists $\bar{\delta} < 1$ s.t. $E [y_\delta] = \frac{x_{n+1}}{2}$ for all $\delta > \bar{\delta}$.

- Suppose $m = n - 1$, so that the majority coalition contains all but one member of the bench. Then, there exists $\bar{\delta} < 1$ s.t. $E [y_\delta] = \frac{1}{2} (x_{m/2}, x_{m+1}/2)$ for all $\delta > \bar{\delta}$. (I.e. the expected policy coincides with the mid-point of the ideal policies of the two medians.)

In this paper, we do not take up the issue of nominations to the bench. However, in concluding this section, we briefly note the stark implications of Proposition 3 for the president’s optimal nomination’s choice. An important implication of the proposition is that equilibrium outcomes depend not only on the relative ordering of the ideal policies of the judges, but their absolute location in state space. The president’s nomination problem is, thus, not simply a ‘move-the-median’ game. The president could nominate two different judges, both occupying the same relative position in the ordering, but with different implications for the equilibrium policies chosen.

## 4 Dispositional Voting

We now consider dispositional voting, which occurs before the bargaining within the dispositional majority. We begin with some terminology. By “utility maximizing voting,” we meant that the judge votes on the disposition to maximize its utility, correctly anticipating the effect of its dispositional vote on the choice of rule. By “sincere voting” we mean that the judge votes strictly in accord with its most preferred rule, without considering the effects of this dispositional vote on the composition of the dispositional majority and hence the content of the opinion. Using the notation defined earlier, sincere voting by judge $j$ corresponds to $d^j = r(z, x^j)$. Sincere dispositional voting may sometimes be compatible with utility maximizing voting. “Sophisticated” dispositional voting means voting “insincerely” at the dispositional stage ($d^j \neq r(z, x^j)$), in order to affect subsequent bargaining over the opinion. So, sophisticated voting means voting “incorrectly” at the dispositional stage in order to join the dispositional majority and participate in bargaining over the opinion’s content.
4.1 First Round Assignment

In the first stage, each judge’s dispositional vote depends on their belief about the equilibrium policies that will result, given differently composed majority coalitions. In the previous section, we determined the policy that each judge would propose, in equilibrium. The policy that will likely be proposed depends on the judge who is selected by the chief justice (or the most senior judge in the majority) to draft the proposal. For each majority coalition $M \subset \{1, \ldots, n\}$, let $s(M, d, z) \in \{1, \ldots, n\}$ denote the judge who is selected to make the first proposal. We must have $s \in M$ – the selected judge must be in the majority coalition. Moreover, let $\gamma(M, d, z) = y_s$ be the policy that the selected judge will propose in equilibrium.

The function $s$ depends on the particular incentives faced by the chief (or most senior) justice. In a naive model, we might suppose that the chief is purely motivated to maximize her utility from the case. But this would imply that the chief justice always assigns the opinion to herself – an assumption at odds with the actual practice of recent chiefs. Indeed, the court has maintained a practice of trying to share the workload of opinion writing amongst its members. Such a policy might be rationalized by noting that opinion writing is costly, and so the chief makes her assignment choice taking into account the associated direct and opportunity costs. Other factors may also be at play. Given the many additional incentives that would need to be incorporated, it is clear that providing micro-foundations for the chief justice’s selection is outside the scope of this paper. Instead, we take a reduced form approach, taking the selection function $s$ as given. We make the following assumption:

Assumption 1. Let $M \subset \{1, \ldots, n\}$ be a majority coalition, and suppose $j \notin M$. Then $|\gamma(M \cup \{j\}) - x^j| \leq |\gamma(M) - x^j|$.

Assumption 1 is a rationality condition, analogous to the weak axiom of revealed preference. It says that introducing a new member to the coalition shouldn’t cause the chief to select a judge who would implement a policy that is worse from $j$’s perspective. To build intuition, suppose $j > \max\{i \in M\}$, so that introducing judge $j$ pulls the center of mass of the coalition to the right. Doing so might cause some previously-feasible left-wing policies to no longer be feasible, which would require the chosen policy to move in $j$’s direction. Moreover, any left-wing policy that can now be implemented, could have been implemented even when $j$ was not in the coalition. So, if she were to switch, one of her choices must have been sub-optimal.
Our reduced form approach is intended to capture a structurally sound decision making problem for the chief. Generically, the chief will not be indifferent between her choices. Moreover, we know that in an extensive form game with perfect information, pure strategy equilibria always exist. Hence, our focus on pure strategies for the chief justice (by assuming her selection is a singleton set) is not overly restrictive. (Additionally, we know that all judges make the same proposal in equilibrium as $\delta \to 1$, and so in this limit, the chief’s selection is inconsequential. Any judge will do.)

## 4.2 Optimal Dispositional Coalitions

Let $M^0(z)$ and $M^1(z)$ be the sets of agents who, if voting sincerely, would choose dispositions ‘0’ and ‘1’, respectively. Let $d^*$ be the disposition of the court, and let $M^*$ be the equilibrium majority coalition.

**Lemma 4.** All judges who agree with the disposition of the court will vote sincerely. Formally, for each $i \in \{0, 1\}$, if $d^* = i$, then $M^i \subseteq M^*$.

Lemma 4 states that, if a judge is not dispositionally pivotal, and a sincere vote would put her in the majority, then she will vote sincerely. The intuition is straightforward: The judge cannot affect the dispositional outcome. If she joins the majority, she can (weakly) draw the equilibrium closer to her ideal policy, whilst avoiding the hedonic cost of being insincere. By contrast, voting strategically, incurs both the explicit cost of being insincere and the opportunity cost of forgoing the ability to affect the policy in the second stage. For these agents, it is a strictly dominant strategy to vote sincerely.

Judges who disagree with the disposition of the court face a more interesting trade-off. Voting strategically enables them to influence the equilibrium proposal, but incurs the cost of insincerity. As we will see through the following results, there will often exist multiple equilibria in the voting game. To understand why, suppose the disposition of the court is $d^* = 1$. Let $M$ be a coalition consisting of at least $k$ judges, with $M^1 \subseteq M$, and $\gamma_M$ be the associated equilibrium policy proposal. Take two judges, $i$ and $j$, who, if they voted sincerely, would find themselves in the minority. It follows that $\gamma_M \leq z < x^i \leq x^j$. We consider two scenarios that illustrate the sources of multiplicity.

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12Note – if we allowed mixed strategies, the results that follow would all continue to hold, provided that the variance in the first round proposal is not too large.
First, suppose \( \gamma(M) = \gamma(M \cup \{i\}) = \gamma(M \cup \{j\}) < \gamma(M \cup \{i, j\}) \). In this scenario, adding one judge to coalition \( M \) has no effect on the equilibrium policy, whereas adding both judges does. (To see why this might be the case, note that adding judges to the coalition changes the identity of the judges whose preferences determine \( y^L \) and \( y^H \). We showed in Lemma \#\# that adding a judge to coalition whose ideal policy is above the mean, will cause both \( y^L \) and \( y^H \) to increase. It might be that adding one judge to the coalition does not shift \( y^L \) sufficiently to change the equilibrium choice, but adding two judges will.) If \( D > 0 \), but is not too large, then the choices for judges \( i \) and \( j \) are strategic complements. Judges \( i \) and \( j \) face a coordination game; they either both want to vote sincerely, or both strategically. As long as the judges are coordinated, their choices are equilibrium consistent.

Second, suppose \( \gamma(M) < \gamma(M \cup \{i\}) = \gamma(M \cup \{j\}) = \gamma(M \cup \{i, j\}) \). In this scenario, adding either judge to coalition \( M \) favorably affects the equilibrium policy offered during the bargaining stage. However, having added one judge, the marginal effect of the adding the second judge is zero. The judges choices are now strategic substitutes (for \( D > 0 \) not too large). They are playing a game of chicken; each wants to vote strategically if and only if the other votes sincerely.

The first type of multiplicity is perverse, in the following sense. Both judges \( i \) and \( j \) would prefer to coordinate on the equilibrium where they vote strategically. Nevertheless, the equilibrium where they both vote sincerely can be sustained by beliefs that that the other judge will vote sincerely. In the most extreme case, when \( D \) is sufficiently small, we can have an equilibrium in which all judges sincerely believe that the disposition should be 0, but all choose 1 believing that all others will do the same. On collegial courts, it is not unreasonable to assume that such beliefs can be dispelled by communication between the judges, and that a coalition of judges can conspire to jointly affect a favorable deviation. To rule out perverse equilibria of this sort, we focus on equilibria that are coalition-proof (see Bernheim, Peleg and Whinston (1987)). An equilibrium fails to be coalition-proof, if there is a mutually beneficial deviation for some subset of agents, such that no sub-subset of those agents then have a further incentive to deviate from the deviation. Whereas Nash equilibria need only survive unilateral deviations, Coalition-proof Nash equilibria (CPNE) must also survive joint deviations by stable coalitions. The notion of coalition-proofness thus refines

\[ \text{In standard bargaining models, the usual refinement is to require that strategies are weakly dominant. This, in effect, requires each player to vote as though they were pivotal. Such a refinement is reasonable, in so far as the players' payoffs are constant under any scenario in which their vote is not pivotal. However, in our model, the agent's will generically have a strict preference between their choices, even if their vote does not change the ultimate dispositional outcome. Hence, the weak-dominance criterion has no bite if applied to overall utilities, and is too strong if applied to dispositional outcomes alone.} \]

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the set of Nash Equilibria, by ruling out equilibria in which a subset of agents are trapped in a situation that is inferior, but which from which they could jointly and stably escape. When strategic complementarity creates multiple equilibria, coalition-proofness selects the equilibrium that is ‘most plausible’, in the sense of ensuring that those complementarities are exploited as far as possible.

Selecting a focal equilibrium from amongst the multiplicity that might arise when judges’ choices are strategic substitutes is less straight-forward. Our approach is motivated by the following result:

**Lemma 5.** Let $M$ be a Nash equilibrium coalition. There exists a connected coalition $M'$ with $\#M' = \#M$ and such that $M'$ is also a Nash equilibrium coalition. Moreover, the $M'$ equilibrium can be sustained over a larger range of value of $D$ than the $M$ coalition.

To make sense of Lemma 5, first note that the net benefit of voting strategically decreases as the ideal policy moves further away from the equilibrium policy. (This becomes evident by examining equations (A3) and (A4) in the appendix.) Hence, more extreme judges should be less inclined to vote strategically than moderate judges. If a majority coalition is disconnected, then a relatively extreme judge finds it optimal to vote strategically, whilst a relatively moderate judge finds it optimal to vote sincerely. Lemma 5 formalizes the intuition, that if this true, the moderate judge must be willing to vote strategically if the extreme one does not. Moreover, the moderate judge should be more willing to do so, in the sense that he would continue to vote strategically even if the cost of insincerity somewhat increased.

Disconnected coalitions arise when judges’ choices are strategic substitutes. For the remainder of this paper, we will focus attention on connected coalitions. Since for every equilibrium coalition that is disconnected, there is a corresponding connected coalition that induces the same disposition and (roughly) the same equilibrium policy, limiting attention to connected coalitions is not substantively restrictive. Additionally, this equilibrium selection mechanism has two desirable features. First, as the second part of Lemma 5 tells us, connected coalitions are equilibrium consistent over a larger range of the cost parameter $D$ than a corresponding disconnected coalition. It follows that connected coalitions are ‘robust’, in the sense that they are least brittle to perturbations in the cost of insincerity. Second, as an empirical matter, coalitions appear for the most part to be connected.\(^{14}\)

\(^{14}\)We again stress, that our focus on connected equilibria is limited to the equilibrium selection problem when multiple equilibria exist. If it were the case that there existed a unique equilibrium that was disconnected, then that necessarily be the equilibrium predicted by our model. We discard disconnected equilibria, only in so far as there is a connected coalition that produces substantially similar equilibrium policies.
We are now ready to characterize the main results in this section.

**Proposition 4.** There exist functions $j^*_0(D, z) \in \{1, ..., k\}$ and $j^*_1(D, z) \in \{k, ..., n\}$, which uniquely define the (connected) majority coalition in a CPNE, as follows:

- Suppose $d^* = 1$. Then $M^* = \{1, ..., j^*_1(D, z)\}$.
- Suppose $d^* = 0$. Then $M^* = \{j^*_0(D, z), ..., n\}$

Moreover, $j^*_0(D, z)$ is (weakly) increasing in $D$ and $j^*_1(D, z)$ is (weakly) decreasing in $D$. As such, $\#M^*$ is weakly decreasing in $D$.

Proposition 4 states one of the main claims of this paper. This is a unique coalition proof Nash equilibrium in which the optimal coalition is connected. The functions $j^*_0$ and $j^*_1$ pin down the composition of this coalition, by identifying the most extreme judge to join the coalition. If $j^*_0 = 1$ or $j^*_1 = n$, then the equilibrium coalition consists of all members of the court, and the disposition is unanimous. Moreover, Proposition 4 highlights the relationship between the cost of insincerity and the size of the majority coalition. As this cost increases, the majority coalition shrinks. The intuition is straight-forward. When $D = 0$, insincerity is costless, and so strategic voting is weakly dominant for every judge. As $D$ increases, the hedonic cost of ruling incorrectly increases faster for extreme agents relative to moderate ones. (This follows since this cost is increasing in the distance of the case from each judge’s ideal policy.) Hence, the most extreme judge will be the first to find strategic voting to be prohibitively costly. As $D$ increases further, the next most extreme judge leaves the coalition, and so on, until the only judges left in the coalition are those voting sincerely. This motivates the following result:

**Corollary 3.** If the cost of insincerity is sufficiently large, then all judges will vote sincerely. The median justice is dispositionally pivotal. (Formally, there exists $\bar{D} > 0$ s.t. whenever $D > \bar{D}$, $d^j = r(z; x^j)$ for each $j$, and $d^* = d^k$.)

The intuition behind Lemma 3 is straight-forward. If $D$ is sufficiently large, then the penalty for voting strategically overwhelms the policy benefit from doing so. All judges will vote sincerely. By construction, the sincere coalitions are connected, and the larger coalition must contain the median judge. Hence, the disposition of the court must coincide with the median judge’s ideal disposition.
In the analysis thus far, we have characterized the optimal dispositional choice of each judge, given their belief about the eventual disposition of the court. We showed, importantly, that judges have an incentive to vote strategically, in order to take part in the bargaining process, and affect the opinion of the court. equilibrium. We complete this section by turning our attention to determining the disposition of the court.

**Proposition 5.** In the unique connected coalition-proof Nash equilibrium, the median justice is dispositionally pivotal.

In the previous section, we showed that the policy of the court need not reflect the ideal policy of the median justice in the majority, or even the median judge on the bench. However, Proposition 5 shows that the median judge is pivotal in determining the disposition of court. The intuition follows immediately from the preceding results. Since they are connected and contain at least a bare majority of judges, every equilibrium coalition must contain the median judge. Hence, the dispositional vote of the median judge must coincide with the disposition of the court.

We stress that, whilst the median justice is pivotal, it need not follow that the disposition of the court coincides with the median judge’s sincere assessment of the case; she may vote strategically. To see when this will happen, let $M_0^*(z,D)$ and $M_1^*(z,D)$ denote the majority coalitions that result if the disposition of the court is 0 and 1, respectively. For concreteness, suppose $x^k < z$, so that the median judge would choose disposition 1 if she voted sincerely. Then, sincerity is optimal provided:

$$1 - (\gamma (M_1^*) - x^k)^2 \geq 1 - (\gamma (M_0^*) - x^k)^2 - D (z - x^k)$$

$$D \geq \frac{2 (\gamma (M_0^*) - \gamma (M_1^*))}{z - x^k} \left[ x^k - \frac{\gamma (M_0^*) + \gamma (M_1^*)}{2} \right]$$

where $\gamma (M_0^*) \geq z \geq \gamma (M_1^*)$. Notice that if $|\gamma (M_0^*) - x^k| \geq |\gamma (M_1^*) - x^k|$, then the right hand term is non-positive, and so sincere voting is optimal for every $D \geq 0$. By contrast, if $|\gamma (M_0^*) - x^k| < |\gamma (M_1^*) - x^k|$, then strategic voting will be optimal provided that the cost of insincerity is not too large. To make sense of this, note that the condition implies that the policy induced by voting strategically is closer to the median judge’s ideal policy than is the policy induced by voting sincerely. For example, this might be the case, if the median judge is slightly left of center, all other leftist judges are on the extreme left, and rightist judges are moderate.

We note that the median judge’s motivation for voting strategically differs from that of the other judges. For non-pivotal judges (i.e. other than the median), the benefit of voting
strategically is that the judge is included in the majority coalition, and may thus direct the
court’s equilibrium policy towards their ideal. By contrast, since she is pivotal, the median
judge will always be in the majority coalition. Her motivation is to vote strategically is
choose amongst equilibrium policies, rather than to purchase the right to participate in the
second-stage bargaining.

<<< Comparative statics on $z$ to be written >>>

5 Conclusion

In this paper, we presented a sequential bargaining model of a multi-member appellate court.
When deciding cases, the court must decide both the case disposition, and a legal rule that
rationalizes the disposition and which provides guidance to lower courts about how to decide
future cases. In our model, these decisions are made sequentially, by majority rule. In the
first stage, the judges cast dispositional votes, with a majority deciding the disposition of
the court. The dispositional vote also determines the subset of judges who participate in the
determination of the court’s legal rule. The policy of the court is determined by bargaining
between the members of the dispositional majority, and requires a majority of the full bench
(rather than merely the dispositional majority). Judges are assumed to have preferences
over both case dispositions and policy rules.

Our framework highlights several important features of judicial decision making. First, since
the dispositional vote acts as a gateway into the policy making stage, each judge may have
an incentive to cast a strategic dispositional vote in the first stage, in order to influence
the subsequent second stage policy. Moreover, the costs and benefits of voting strategically
depend upon the nature of the case to be decided. This is true for two reasons. First, since
the court’s announced policy must be consistent with its disposition, there are limits to the
amount by which a strategically voting judge may ‘moderate’ the court’s policy. Second,
dispositional preferences are assumed to satisfy the increasing differences in dispositional
values property, which causes the immediate cost of voting strategically to depend on
the case being decided. Judges may more profitably vote strategically if the case appears
‘contestable’ from their perspective, than if it appears clear cut.

We show that, in equilibrium, the median judge is pivotal over case dispositions. Further-
more, we show that equilibrium coalitions are connected — meaning that the most extreme
judges are the least likely to vote strategically. By contrast, moderate judges may frequently
find themselves voting contrary to their preferred outcome, in order to affect the court’s policy outcome.

Second, the sequential structure of our game highlights that, although policy making requires a simple majority of the entire bench, when dispositional majorities are non-unanimous, the dispositional majority faces an effective super-majority requirement. We characterize the equilibria of unidimensional spatial bargaining games under conditions under for any (super)-majority condition. These equilibria will generically depend upon the ideal policies of the agents in the dispositional majority as well as the location of the case. A important novelty of this paper is in characterizing the limit equilibria of the bargaining game as $\delta \to 1$, which we interpret as the limit as the cost of proposing counter-proposals becomes arbitrarily small — an assumption which we think is reasonable given the institutional setting of the court. We show that, in the limit, it as if the dispositional majority endogenously separates into two factions. The announced policy is the either the ideal policy of some pivotal judge (not necessarily the median of the dispositional majority), or the result of asymmetric Nash Bargaining between representative leaders of the factions, with bargaining weights proportional to factional size. Importantly, in the limit, the chosen policy will never coincide with the ideal policy of the median judge — and so whilst the median judge decides the disposition of the court, she does not also determine the policy of the court. Our result thus stands in contrast to both median voter results and median-of-the-majority results that have been proposed in the existing literature.

Relatedly, our analysis provides microfoundations for the emergence of policy coalitions within the court — which gives a theoretical basis to commonly discussed notions of court separating into left- and right-wing blocs.

Finally, our model highlights the importance of the case-space approach to modelling courts, by show-casing the importance of case location in both the composition of dispositional majorities (and the likelihood of strategic voting) and the equilibrium policies that result.

6 Appendix

Proof of Lemma 1. We prove this lemma by stating and proving a number of intermediate claims.

Claim 1: Let $y$ be a generic policy. If $|y - x^i| \leq |E[y] - x^i|$, then $i$ will support proposal $y$. 

To see this, note that:

\[ u(y; x^i) = 1 - (y - x^i)^2 \]
\[ \geq 1 - (E[y] - x^i)^2 \]
\[ \geq 1 - \left[ (E[y] - x^i)^2 + \text{Var}(y) \right] \]
\[ = v^i \]
\[ \geq \delta v^i \]

Claim 2: If \( x^i < E[y] \), then \( y^i \in [x^i, E[y]] \). If \( x^i > E[y] \), then \( y^i \in [E[y], x^i] \).

To see this, note that offering \( y^i = E[x] \) guarantees unanimous support. Hence, the most any agent would need to compromise to is \( E[x] \). (To see that there is never an incentive to offer \( y^i < x^i \), note by claim 1 that every judge with \( x^j \leq x^i \) will support any policy \( y \in [x^i, E[y]] \). Moreover, any judge with \( x^j > x^i \) that would support \( y < x^i \), would also support \( y = x^i \). Hence, if \( y < x^i \) can achieve majority support, so must \( y = x^i \), and this is clearly preferred by judge \( i \).)

Claim 3: If \( x^i < E[y] \), then, in equilibrium, \( i \) will receive the support of all judges \( j < i \). Similarly, if \( x^i > E[y] \), then \( i \) will receive the support of all judges \( j > i \).

This follows from claims 1 and 2. WLOG, take the case of \( x^i < E[y] \). By claim 2 \( x^i \leq y^i \leq E[y] \). Hence, for any \( j < i \), \( |y^i - x^j| \leq |E[y] - x^j| \), and so by claim 1, \( j \) prefers \( y^i \) to the continuation game.

Claim 4: Suppose either: (i) \( y < x^i < x^j \) and \( y < E[y] \), or (ii) \( x^j < x^i < y \) and \( E[y] < y \). Then if \( i \) rejects \( y \) so will \( j \).

To see this, WLOG, take case (i). Since \( i \) rejects \( y \), it must be that:

\[ 1 - (y - x^i)^2 < \delta - \delta \left[ (E[y] - x^i)^2 + \text{Var}(y) \right] \]
\[ (1 - \delta) + \delta \text{Var}(y) < (y - x^i)^2 - \delta (E[y] - x^i)^2 \]
Now, for generic $x$, let $\phi(x) = (y - x)^2 - \delta (E[y] - x)^2$. Then

$$
\phi'(x) = 2\delta (E[y] - x) - 2(y - x) = 2[(1 - \delta)x + \delta E[y] - y]
$$

By condition (i), $x > y$ and $E[y] > y$, and so $(1 - \delta)x + \delta E[y] > y$. Hence $\phi'(x) > 0$. It follows that: $(1 - \delta) + \delta \text{Var}(y) < (y - x^i)^2 - \delta (E[y] - x^i)^2 < (y - x^i)^2 - \delta (E[y] - x^i)^2$, and so $j$ rejects $y$.

We are now ready to prove the lemma. Claims 4 implies that the equilibrium coalition will be connected. Claim 3 implies that all judges on one side of the proposing judge (the side further from the expected policy) will join the coalition. An equilibrium proposal must have the support of at least $k$ judges. The lemma follows.

\textbf{Proof of Proposition 1}. First, we characterize the optimal proposal for each judge. Let $y^L$ and $y^H$ be as defined in the proposition. Notice that $y^L \leq E[y] \leq y^H$. (To see this, first note that since $b \leq y^i \leq \bar{y}$ for each $i$, it follows that $b \leq E[y] \leq \bar{b}$. Next, $|E[y] - y^k| \leq \sqrt{(1 - \delta) + \delta (E[y] - y^k)^2 + \delta \text{Var}(y)}$, with strict equality only if $\delta = 1$ and $\text{Var}(y) = 0$. Hence, $E[y] \geq x^k - \sqrt{(1 - \delta) + \delta (E[y] - x^k)^2 + \delta \text{Var}(y)} = y^L$. A similar argument shows that $E[y] \leq y^H$.)

Let $E[y]$ be the expected equilibrium proposal, and take some judge $i$ with $x^i < E[y]$. By Lemma 1 we know that $y^i \in [x^i, E[y]]$ and $\{1, ..., k\} \subset C^i$. Hence $y^i$ must be attractive enough for judge $k$ to support. This implies:

$$
1 - (y^i - x^k)^2 \geq \delta \left[1 - (E[y] - x^k)^2 - \text{Var}(y)\right]
$$

$$
|y^i - x^k| \leq \sqrt{(1 - \delta) + \delta (E[y] - x^k)^2 + \delta \text{Var}(y)}
$$

This requires $y^i \geq x^k - \sqrt{4(1 - \delta) + \delta (E[y] - x^k)^2 + \delta \text{Var}(y)}$. Feasibility, requires $y^i \geq b$. Hence, if $x^i \geq y^L$, it is optimal for judge $i$ to offer $y^i = x^i$. Otherwise, she will offer $y^i = y^L$.

Now, we show that the equilibrium exists and is unique. To do so, I must show that there is a unique pair $(y^L, y^H)$ that satisfy (1) and (2) in the proposition. It will prove useful to consider a change of variable, by considering the distances $\xi^L = x^k - y^L \geq 0$ and
\( \xi^H = y^H - x^{m-k+1} \geq 0 \). Let \( F \) be the set of bounded functions \( f : [0, 1] \times [-1, 1]^m \rightarrow [0, 2] \). Consider the operator \( T : F^2 \rightarrow F^2 \) defined by: \( T = (T_L, T^H) \) where:

\[
T_L (\xi^L, \xi^H) = \sqrt{(1 - \delta) + \delta \frac{1}{m} \sum_i \left( \min \left\{ \max \left\{ x_i - x_k, -\xi^L \right\}, \xi^H - (x_k - x_{m-k+1}) \right\} \right)^2}
\]

\[
T^H (\xi^L, \xi^H) = \sqrt{(1 - \delta) + \delta \frac{1}{m} \sum_i \left( \min \left\{ x_i - x_{m-k+1}, (x_k - x_{m-k+1}) - \xi^L \right\}, \xi^H \right)^2}
\]

where \( \xi^L (\delta, x^1, ..., x^m), \xi^H (\delta, x^1, ..., x^m) \in F \).

We show that \( T \) satisfies Blackwell’s conditions. Suppose \( (\xi^L_1, \xi^H_1) \leq (\xi^L_2, \xi^H_2) \).

Since \( \min \left\{ \max \left\{ x_i - x_k, -\xi^L \right\}, \xi^H - (x_k - x_{m-k+1}) \right\} \) is weakly decreasing in \( \xi^L \) and weakly increasing in \( \xi^H \), then \( T_L (\xi^L_1, \xi^H_1) \leq T_L (\xi^L_2, \xi^H_2) \), and similarly for \( T^H \). This confirms monotonicity. To show discounting, first note that:

\[
\frac{\delta}{m} \sum_i \left( \min \left\{ x_i - x_k, -\left( \xi^L + c \right) \right\}, \left( \xi^H + c \right) - (x_k - x_{m-k+1}) \right)^2 \leq \frac{\delta}{m} \sum_i \left( \min \left\{ x_i - x_k, -\xi^L \right\}, \xi^H - (x_k - x_{m-k+1}) \right)^2 + c^2
\]

Next, note generically, by the mean value theorem, there exists \( \lambda \in (0, 1) \) such that:

\[
\sqrt{a + b \frac{1}{m} \sum_i (|x_i| + c)^2} = \sqrt{a + b \frac{1}{m} \sum_i x_i^2 + b \frac{1}{m} \sum_i (|x_i| + \lambda c) \frac{c}{\sqrt{a + b \frac{1}{m} \sum_i (|x_i| + \lambda c)^2}}} = \sqrt{a + b \frac{1}{m} \sum_i x_i^2 + \kappa c}
\]

where \( \kappa = \frac{1}{\sqrt{a + b \frac{1}{m} \sum_i (|x_i| + \gamma c)}} \in (0, 1) \). Applying this to our operator gives:

\[
T_L (\xi^L + c, \xi^H + c) \leq \sqrt{(1 - \delta) + \delta \frac{1}{m} \sum_i \left( \min \left\{ x_i - x_k, -\xi^L \right\}, \xi^H - (x_k - x_{m-k+1}) \right)^2 + c^2}
\]

\[
\leq \sqrt{(1 - \delta) + \delta \frac{1}{m} \sum_i \left( \min \left\{ x_i - x_k, -\xi^L \right\}, \xi^H - (x_k - x_{m-k+1}) \right)^2 + \kappa c}
\]

\[
= T_L (\xi^L, \xi^H) + \gamma c
\]

which proves discounting. (Note, this last step requires \( \delta < 1 \), which ensured that our generic \( a > 0 \).) Since \( T \) satisfies Blackwell’s conditions, it is a contraction mapping, and so admits
a unique fixed point.

To prove continuity, let \( F_C \subset F \) be the set of continuous and bounded functions. It suffices to show that \( T : F_C \to F_C \). This follows immediately by the continuity of the polynomials. \( \square \)

**Proof of Lemma 2.** Let \( \delta < 1 \). Suppose \( y_\delta^H = y_\delta^L \). This implies that \( y^j = y^L = y^H \) for all \( j = 1, \ldots, m \), and so \( E[y] = y^H = y^L \) and \( \text{Var}(y) = 0 \). But then, by part 2 of Proposition 1, \( E[y] - x^k = -\sqrt{1 - \delta + \delta (E[y] - x^k)^2} \) and so \( E[y] = x^k - 1 \). (This makes use of the fact that \( E[y] \in [x^{m-k+1}, x^k] \), since \( y^j \in [x^{m-k+1}, x^k] \) for all \( j \).) By a similar argument, \( E[y] = x^{m-k+1} + 1 \). But, then \( x^k - x^{m-k+1} = 2 \), which cannot be since \( 0 \leq x^{m-k+1} \leq x^k \leq 1 \). Hence, \( y^H > y^L \). Finally, we show that \( \text{Var}(y) \to 0 \). This follows by the fact that the equilibrium correspondence is upper-hemicontinuous at \( \delta = 1 \), and that \( \text{Var}(y) = 0 \) for every equilibrium at \( \delta = 1 \). (We prove this in the following lemma.) \( \square \)

**Proof of Lemma 3.** Let \( \delta = 1 \). Suppose there is a no-delay equilibrium. First, we show that \( \text{Var}(y) = 0 \). Suppose \( y^L < y^H \). Then

\[
(x^k - y^L)^2 = \frac{1}{m} \sum_j (x^k - y^j)^2 \leq \frac{1}{m} \sum_{x^l < E[y]} (x^k - y^l)^2 + \frac{1}{m} \sum_{x^j \geq E[y]} (x^k - y^H)^2 < (x^k - y^L)^2
\]

where the final inequality follows from the fact that \( |x^k - y^L| > |x^k - y^H| \), by construction. But this is obviously inconsistent. Hence \( y^L = y^H \), and so all judges propose \( y^j = y^L = y^H = E[y] \). Hence \( \text{Var}(y) = 0 \). Next, conjecture an equilibrium in which all agent’s offer \( y^j = y \). Judge \( j \) will accept any \( y' \) s.t. \( |y' - x^j| \leq |y - x^j| \). Hence judge \( j \)’s acceptance set is: \( A^j = \{y' | y' \in [y, 2x^j - y]\} \) if \( x^j > y \) and \( A^j = \{y' | y' \in [2x^j - y, y]\} \) if \( x^j \leq y \). The set of proposals that will be accepted is \( A = \{y' | y' \in A^j \text{ for } k \text{ judges}\} \). Notice that \( y \in A^j \) for every \( j \), and so \( y \in A \). Suppose \( y < x^{m-k+1} \). Then \( y' \in [y, x^{m-k+1}] \) will be acceptable to judges \( m - k + 1, \ldots, m \), and so \( [y, x^{m-k+1}] \in A \). Moreover, any such offer is a profitable deviation to each of these judges, and so they each have an incentive to deviate from the equilibrium. Hence \( y \not< x^{m-k+1} \). A similar argument shows that \( y \not< x^k \). Finally, suppose \( y \in [x^{m-k+1}, x^k] \). Then, for any other policy \( y' \), it cannot be that \( y' \) is closer to the ideal policies of at least \( k \) judges. Hence \( A = \{y\} \), which is equilibrium consistent. \( \square \)

**Proof of Lemma 3.** Let \( i^* = \min \{i | x^i \geq b^{i+1}\} \) and let \( \mu = \lim_{\delta \to 1} E[y_\delta] \).
For some arbitrary \( i \in \{1, \ldots, m-1\} \), suppose \( \mu \in (x^{i-1}, x^i) \). Since \( \text{Var}\ (y_\delta) \to 0 \), there exists \( \bar{\delta} < 1 \), such that for \( \delta > \bar{\delta} \), \( x^{i-1} < y^H_\delta < \mu < y^H_\delta < x^i \). Then \( E[y_\delta] = x^{i-1} + \frac{i-1}{m} y^H_\delta + \frac{m+1-i}{m} y^L_\delta \).

Let \( \varepsilon^L_\delta = E[y_\delta] - y^L_\delta \) and \( \varepsilon^H_\delta = y^H_\delta - E[y_\delta] \). Then \( \varepsilon^H_\delta = \frac{i-1}{m+1-i} \varepsilon^L_\delta \). Moreover, \( \text{Var}\ (y_\delta) = \frac{i-1}{m} (\varepsilon^L_\delta)^2 + \frac{m+1-i}{m} (\varepsilon^H_\delta)^2 = \frac{i-1}{m+1-i} (\varepsilon_\delta^L)^2 \).

Now, by definition: \((x^k - y^L_\delta)^2 = 1 - \delta + \delta (x^k - E[y_\delta])^2 + \delta \text{Var}\ (y_\delta)\). Using the fact that 
\((x^k - y^L_\delta)^2 = (x^k - E[y_\delta])^2 + 2(x^k - E[y_\delta])\varepsilon^L_\delta + (\varepsilon^L_\delta)^2\), this implies:

\[
\begin{equation}
(1 - \delta) (x^k - E[y_\delta])^2 + 2\varepsilon^L_\delta (x^k - E[y_\delta]) = 1 - \delta + (\varepsilon^L_\delta)^2 \left( \frac{i-1}{m+1-i} - 1 \right)
\end{equation}
\]

Equations (1) and (2) implicitly define \( \varepsilon^L_\delta \) and \( E[y_\delta] \) in terms of \( \delta, x^k, x^{m-k+1} \) and \( \frac{i-1}{m+1-i} \).

Moreover, since the equations are twice continuously differentiable, the implicit function theorem implies that \( \frac{\partial \varepsilon^L_\delta}{\partial \delta} \) (and all other partial derivatives) must be continuously differentiable.

Totally differentiating gives:

\[
\begin{bmatrix}
2 (x^k - E[y_\delta]) - 2 \varepsilon^L_\delta (i \cdot \frac{i-1}{m+1-i} - 1) \\
2 (m+1-i) (E[y_\delta] - x^{m-k+1}) - 2 \varepsilon^L_\delta (i \cdot \frac{i-1}{m+1-i})
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \varepsilon^L_\delta}{\partial E[y_\delta]} \\
\frac{\partial E[y_\delta]}{\partial \delta}
\end{bmatrix}
\begin{bmatrix}
2 (x^k - E[y_\delta]) - 2 \varepsilon^L_\delta (i \cdot \frac{i-1}{m+1-i}) \\
2 (m+1-i) (E[y_\delta] - x^{m-k+1}) - 2 \varepsilon^L_\delta (i \cdot \frac{i-1}{m+1-i})
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \varepsilon^L_\delta}{\partial \delta} \\
\frac{\partial E[y_\delta]}{\partial \delta}
\end{bmatrix}
\]

and taking the limit as \( \delta \to 1 \) gives:

\[
\begin{bmatrix}
2 (x^k - \mu) \\
2 \frac{i-1}{m+1-i} (\mu - x^{m-k+1})
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \varepsilon^L_\delta}{\partial \delta} \\
\frac{\partial E[y_\delta]}{\partial \delta}
\end{bmatrix}
\begin{bmatrix}
2 (x^k - \mu) \\
2 \frac{i-1}{m+1-i} (\mu - x^{m-k+1})
\end{bmatrix}
\begin{bmatrix}
-1 + (x^k - \mu)^2 \\
-1 + (\mu - x^{m-k+1})^2
\end{bmatrix}
\]

This implies that \( \frac{\partial \varepsilon^L_\delta}{\partial \delta} \to \frac{(x^k - \mu)^2 - 1}{2(x^k - \mu)} = \frac{i-1}{m+1-i} \frac{(\mu - x^{m-k+1})^2 - 1}{2(\mu - x^{m-k+1})} \). If \( \mu \) is the limit policy, then it solves the equation:

\[
(3)
\frac{(x^k - \mu)^2 - 1}{2(x^k - \mu)} = \frac{i-1}{m+1-i} \frac{(\mu - x^{m-k+1})^2 - 1}{2(\mu - x^{m-k+1})}
\]

Next, consider the Nash Bargaining problem:

\[
\max_b \left[ 1 - (x^{m-k+1} - b)^2 \right]^\frac{i-1}{m} \left[ 1 - (x^k - b)^2 \right]^\frac{m+1-i}{m}
\]

It is easy to see that the first order conditions implied by this problem coincides with (3).
Hence, if \( \mu \) is the limit policy of the bargaining game, it must coincide with the asymmetric Nash Bargaining solution.

Now, by assumption, \( x^{i-1} < b^i < x^i \). Notice that \( x^i \) is increasing in \( i \), whilst \( b^i \) is decreasing. Since \( i^* = \min \{ i | x^i \geq b^{i+1} \} \), it follows that \( i \geq i^* \). If \( i > i^* \), then \( b^{i+1} < b^i \leq x^{i-1} \leq x^i \), which contradicts the assumption. Hence \( i = i^* \). Consistency then requires that \( x^{i^*} > b^{i^*} \).

Next, we show that if \( \mu = \frac{1}{2} \left( x^{m-k+1} + x^k \right) \), then there exists \( \tilde{\delta} > 1 \), such that \( E [y_\delta] = \mu \) for all \( \delta > \tilde{\delta} \). Note that there are two conditions under which this can arise. First, if \( m \) is even, then \( b^{m+1} = \frac{1}{2} \left( x^k + x^{m-k+1} \right) \), and so the condition is satisfied if \( i^* = \frac{m}{2} + 1 \) and \( x^m > \frac{1}{2} \left( x^k + x^{m-k+1} \right) \). (To see why \( b^{m+1} = \frac{1}{2} \left( x^k + x^{m-k+1} \right) \), note that \( b^{m+1} \) is the maximizer of \( \left[ 1 - (x^{m-k+1} - b)^2 \right]^{\frac{1}{2}} \left[ 1 - (x^k - b)^2 \right]^{\frac{1}{2}} \), which clearly occurs at the midpoint.)

Second, this can arise if, by coincidence, \( x^{i^*+1} = \frac{1}{2} \left( x^{m-k+1} + x^k \right) \).

In the first case, take \( \delta \) sufficiently large, so that \( x^m < y^L_\delta < \mu < y^H_\delta < x^m \). Since \( j = m - j \), equations (1) and (2) reduce to:

\[
(1 - \delta) \left( x^k - E [y_\delta] \right)^2 + 2\varepsilon^L_\delta \left( x^k - E [y_\delta] \right) = 1 - \delta
\]

\[
(1 - \delta) \left( E [y_\delta] - x^{m-k+1} \right)^2 + 2\varepsilon^L_\delta \left( E [y_\delta] - x^{m-k+1} \right) = 1 - \delta
\]

which implies that:

\[
\varepsilon^L_\delta = \frac{1 - \delta}{2} \cdot \frac{1 - (x^k - E [y_\delta])^2}{(x^k - E [y_\delta])} = \frac{1 - \delta}{2} \cdot \frac{1 - (E [y_\delta] - x^{m-k+1})^2}{(E [y_\delta] - x^{m-k+1})}
\]

This implies that \( x^k - E [y_\delta] = E [y_\delta] - x^{m-k+1} \), and so \( E [y_\delta] = \frac{1}{2} \left( x^{m-k+1} + x^k \right) \). And this is true, not only in the limit, but for every \( \delta \) sufficiently large. A similar argument holds in the second case.

**Proof of Lemma 4.** Let \( z \) be an arbitrary case. Suppose \( d^* = 0 \). (The other case is analogous.) Then \( M^0 = \{ j | x^j > z \} \). Moreover, all feasible second stage policies must satisfy \( y \geq z \). Suppose there is a \( j \), such that \( j \in M^0 \) and \( j \notin M^* \). Then

\[
E \left[ u^j \left( d^i = 1; d^{-j} \right) \right] > E \left[ u^j \left( d^i = 0; d^{-j} \right) \right]
\]

\[
1 - \left( \gamma_M - x^j \right)^2 - D |x^j - z| > 1 - \left( \gamma_{M \cup \{j\}} - x^j \right)^2
\]

\[
-D |x^j - z| > \left( \gamma_M - x^j \right)^2 - \left( \gamma_{M \cup \{j\}} - x^j \right)^2
\]

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By assumption 1, $|\gamma_{M* \cup \{j\}} - x^j| \leq |\gamma_{M*} - x^j|$, which implies that the RHS term is non-negative. By contrast, the LHS term is strictly negative whenever $D > 0$, which is a contradiction. Hence $j \in M*$.

Proof of Lemma 5. Let $M$ be an equilibrium coalition, and suppose $M$ is not connected. WLOG, suppose $d^* = 1$, so that, by Lemma 4, $M^1(z) \subset M$. Since $M^1$ is a connected coalition and $M$ is disconnected, $M$ must contain members of $M^0(z)$. Then there exists $i < j$ with $i, j \in M^0(z)$, $i \notin M$ and $j \in M$. Then $z < x^i \leq x^j$. Let $M'$ be identical to $M$ except that judge $j$ is replaced by judge $i$. Using arguments made previously, suppose $\gamma_M = \gamma_{M'}$. (Intuitively, judge $i$ and $j$ cannot bring their ideal policy to bear. If recognized, they will each select $y^H$, and so the chief should be indifferent between selecting either to propose. Moreover, each judge affects the equilibrium policies offered by other judges only in so far as their presence affects the relative ordering of coalition members, and they both affect this ordering in the same way.) Since $M$ is an equilibrium, it must be that:

\begin{align*}
(4) \quad 1 - (\gamma_M - x^i)^2 - D(x^i - z) &\geq 1 - (\gamma_{M-\{j\}} - x^j)^2 \\
and:
(5) \quad 1 - (\gamma_M - x^i)^2 &\geq 1 - (\gamma_{M \cup \{i\}} - x^i)^2 - D(x^i - z)
\end{align*}

Clearly, (4) requires $\gamma_{M-\{j\}} < \gamma_M$. Then, since $x^i \leq x^j$, it follows that:

\begin{align*}
1 - (\gamma_{M'} - x^i)^2 - D(x^i - z) &\geq 1 - (\gamma_{M'-\{i\}} - x^i)^2
\end{align*}

noting that $\gamma_{M'} = \gamma_M$ and that $M' - \{i\} = M - \{j\}$ by construction. Moreover, if $x^i < x^j$, then the inequality is strict, and continues to be so for some $D' > D$ and even for some $\gamma_{M'} < \gamma_M$. Moreover, since $x^j > x^i$, it follows that $1 - (\gamma_{M'} - x^j)^2 \geq 1 - (\gamma_{M' \cup \{j\}} - x^j)^2 - D(x^j - z)$, and again this inequality is strict if $x^i < x^j$. This confirms that $M'$ is also an equilibrium coalition. We can iterate in this way until we arrive at a connected coalition. □

Proof of Proposition 4. Let $z$ be an arbitrary case. Consider a CPNE with $d^* = 1$. (The other scenario is analogous.) We proceed iteratively. Begin with smallest connected coalition
that is a candidate to be an equilibrium, \( M_\ast \). This is an equilibrium if, for every \( j \in M_\ast \):

\[
1 - \left( \gamma_{M_\ast} - x^j \right)^2 \geq 1 - \left( \gamma_{M_\ast+1} - x^j \right)^2 - D \left( x^j - z \right)
\]

\[
x^j \geq \frac{\gamma_{M_\ast} + \gamma_{M_\ast+1}}{2} + \frac{D}{D - 2 \left( \gamma_{M_\ast} - \gamma_{M_\ast+1} \right)} \left( z - \frac{\gamma_{M_\ast} + \gamma_{M_\ast+1}}{2} \right)
\]

There are two possibilities. If this condition holds, then \( M_\ast \) is a Nash equilibrium coalition of the game (although not necessarily CPNE). Stop. If this condition does not hold, then \( M_\ast \) is not equilibrium consistent. Then, consider the coalition \( M_\ast+1 \). We have just shown that no \( j \in M_\ast+1 \) has a favorable deviation to \( d^j = 0 \). Again take \( j \in M_\ast+1 \). Iterate this procedure, until the condition is satisfied for every \( j \) in the candidate set. If the condition is not satisfied for every \( i \in \{j^\ast, \ldots, n-1\} \), then we have showed that no \( j \in M_n \) will want to deviate from choosing \( d^j = 1 \), and so \( M_n \) is equilibrium consistent.

So far, we have found the smallest equilibrium consistent connected coalition. Let \( M_\ast \) be this coalition. There may be larger equilibrium consistent coalitions as well. Starting from coalition \( M_\ast \), consider joint deviations to coalitions \( M_\ast+2, M_\ast+3, \ldots \) etc. until the first mutually beneficial deviation is found. If no such deviation exists, then \( M_\ast \) is a CPNE. If a beneficial deviation is found, then repeat the procedure, considering joint deviations from this new coalition. The CPNE is the largest coalition for which no additional joint deviations exist.

The comparative statics on \( D \) follow immediately by noting that the RHS of the above equation is decreasing in \( D \).

\[\square\]

**Proof of Lemma ??**. Consider a case \( z \), and suppose WLOG that \( \#M^1(z) > \#M^0(z) \). Since \( M^1 \) and \( M^0 \) are connected sets, this implies that \( k \in M^1 \) and \( x^k < z \). Let \( j^\ast = \min \{ j | j \in M^0 \} \). Conjecture that in equilibrium, \( d^\ast = 1 \) and \( M^\ast = M^1 \). Then By Lemma 4, we know that there is no profitable deviation for any judge \( j \in M^1 \). Now, consider any other coalition \( M' \) satisfying \( M^1 \subset M' \). Relative to \( M^\ast \), \( M' \) involves a (mutual) deviation by the agents in \( M' \cap M^0 \) to vote strategically. Such a deviation is profitable provided that, for each \( j \in M' \cap M^0 \):

\[
1 - \left( \gamma_{M'} - x^j \right)^2 - D \left( x^j - z \right) > 1 - \left( \gamma_{M^\ast} - x^j \right)^2
\]

\[
-\gamma_{M'}^2 + 2\gamma_{M'} x^j - D x^j + Dz > -\gamma_{M^\ast}^2 + 2\gamma_{M^\ast} x^j
\]

\[
[2 \left( \gamma_{M'} - \gamma_{M^\ast} \right) - D] x^j > \gamma_{M'}^2 - \gamma_{M^\ast}^2 - Dz
\]
Assuming $D > 2(\gamma_{M'} - \gamma_{M^*})$, gives:

$$x^j < \frac{\gamma_{M'}^2 - \gamma_{M^*}^2}{2(\gamma_{M'} - \gamma_{M^*}) - D} - \frac{D}{2(\gamma_{M'} - \gamma_{M^*}) - D}z$$

$$= \frac{\gamma_{M'} + \gamma_{M^*}}{2} + \frac{D}{D - 2(\gamma_{M'} - \gamma_{M^*})} \left(z - \frac{\gamma_{M'} + \gamma_{M^*}}{2}\right) = x^* (D, M')$$

Now, if $x^* (D, M') \leq j^*$, then no member of $M^0$ will find the deviation profitable. This will be the case if:

$$1 - (\gamma_{M'} - x^*)^2 - D(x^* - z) \leq 1 - (\gamma_{M^*} - x^*)^2$$

$$D \geq \frac{2(\gamma_{M'} - \gamma_{M^*})}{x^* - z} \left(x^* - \frac{\gamma_{M'} + \gamma_{M^*}}{2}\right)$$

Since we need this to be true for every $M'$, and since $\gamma_{M'} \leq \gamma_{\{1,\ldots,n\}}$ for every $M'$, it suffices to take $D \geq \bar{D} = \frac{2(\gamma_{\{1,\ldots,n\}} - \gamma_{M^*})}{x^* - z} \left(x^* - \frac{\gamma_{\{1,\ldots,n\}} + \gamma_{M^*}}{2}\right) > 0$ (unless $\gamma_{\{1,\ldots,n\}} = \gamma_{M^*}$).

We have considered coalitional deviations in which the disposition of the court does not change. It remains to show that no profitable deviations exist in which the disposition flips. Suppose there is, and let the deviation be to $M'$. Take any judge $j \in M^1 \cap M'$ – such a judge switches his vote to join the opposite dispositional majority. This requires:

$$1 - (\gamma_{M'} - x^j)^2 - D(x^j - z) > 1 - (\gamma_{M^*} - x^j)^2$$

$$D < \frac{2(\gamma_{M'} - \gamma_{M^*})}{z - x^j} \left(\frac{\gamma_{M'} + \gamma_{M^*}}{2} - x^j\right)$$

as before. To prevent such a deviation, it suffices to take $D$ large enough.
References

URL: http://www.jstor.org/stable/2586381


